STATISTICAL METHODS FOR FINANCE: EXAMINATION 2015-16

Three hours; six questions, do five.

Q1. (i) Describe briefly the main contributions of Markowitz's work to mathematical finance.
(ii) Describe briefly the elliptically contoured model, and specify its parametric part and its non-parametric part.
(iii) What are the principal deficiencies of normal (Gaussian) models in mathematical finance?
(iv) How does the asset return distribution depend on the return period?

Q2. Define the *log-normal distribution* $LN(\mu, \sigma)$ with parameters μ and σ. Show that it has mean $\exp\{\mu + \frac{1}{2}\sigma^2\}$.

Describe briefly how the log-normal distribution occurs in mathematical finance.

For a normal distribution $N(\mu, \sigma^2)$ with σ known, obtain a uniformly most powerful test for the simple null hypothesis H_0: $\mu = \mu_0$ against the composite alternative hypothesis H_1: $\mu < \mu_0$.

Q3. The heights of two towers A and B are measured, and so is the difference of their heights (B is the higher). The true values are a and b; the readings are y_A, y_B, y_{B-A}. Formulate a regression model.

Find
(a) the design matrix A, $C := A^T A$, $C^{-1}A^T$,
(b) the projection matrix $P := AC^{-1}A^T$ and $I - P$,
(c) the parameter estimates,
(d) the fitted values,
(e) the ranks of P and $I - P$.
(f) How would you find an unbiased estimator of the error variance σ^2?

Q4. In a sample of size n, a response variable y has two linear predictor variables u and v. Construct the regression plane of y on u, v.

When is this plane unique?

Describe briefly the financial applications of this.

Q5. Define an *autoregressive* time-series model of *order* p, $AR(p)$.

Derive the *Yule-Walker equations*, and show how to solve them.

In the model

$$X_t = \frac{1}{3}X_{t-1} + \frac{2}{9}X_{t-2} + \epsilon_t, \qquad (\epsilon_t) \quad \text{white noise}, \tag{1}$$

find the autocorrelation function $\rho(k)$.

Q6. State without proof Edgeworth's theorem for the density of the multinormal law $N(\mu, \Sigma)$, in terms of the concentration matrix $K := \Sigma^{-1}$.

If a multinormal vector x is partitioned into x_1 and x_2, with μ, Σ, K partitioned accordingly, derive the conditional distribution of x_1 given x_2 in terms of μ, K.

Give the same result in terms of μ and Σ.

(You may assume the formula for the inverse of a partitioned matrix:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} M & -MBD^{-1} \\ -D^{-1}CM & D^{-1} + D^{-1}CMBD^{-1} \end{pmatrix}, \quad M := (A - BD^{-1}C)^{-1},$$

when all inverses exist.)

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