STATISTICAL METHODS FOR FINANCE: EXAMINATION 2016-17

Three hours; six questions, do five.

Q1. In a parametric model, define the *score function* and the *information per reading.*

In a normal model with mean μ known,
(i) find the information per reading on the variance $v := \sigma^2$, and hence the Cramér-Rao bound;
(ii) find an efficient estimator for σ^2.

Q2. Define the *lognormal distribution* $LN(\mu, \Sigma)$ with parameters μ and σ. Show that it has mean $\exp\{\mu + \frac{1}{2}\sigma^2\}$.

Describe briefly how the lognormal distribution occurs in mathematical finance.

For a normal distribution $N(\mu, \sigma^2)$ with σ known, obtain a uniformly most powerful test for the simple null hypothesis $H_0 : \mu = \mu_0$ against the composite alternative hypothesis $H_1 : \mu < \mu_0$.

Q3. (i) Describe briefly the main contributions of Markowitz's work to mathematical finance.
(ii) Describe briefly the elliptically contoured model, and specify its parametric part and its non-parametric part.
(iii) What are the principal deficiencies of normal (Gaussian) models in mathematical finance?
(iv) How does the asset return distribution depend on the return period?

Q4. In a multivariate setting, define the *sample mean vector* $\bar{x}$ and the *sample covariance matrix* $S = S_x$.

For the multivariate normal model $N(\mu, \Sigma)$,
(i) express the likelihood in terms of $\bar{x}$ and S;
(ii) show that $(\bar{x}, S)$ is sufficient for (μ, Σ).

Q5. In an $ARMA(1, 1)$ model

$$X_t = \phi X_{t-1} + \epsilon_t + \theta \epsilon_{t-1},$$

(i) state (without proof) the conditions for stationarity and invertibility.

(ii) Assuming these, find
(a) the variance $\gamma_0 = var\ X_0$;
(b) the covariance $\gamma_k = cov(X_t, X_{t-k})$.

Q6. We are to estimate the parameter θ of a Poisson distribution, using as prior for θ the Gamma distribution $\Gamma(a, b)$ and a sample of size n.
(i) Find the posterior for θ.
(ii) Find its mean, and interpret the posterior mean as a weighted average of the prior mean and the sample mean.