

SMF PROBLEMS 5. 6.11.2017

Q1. The $AR(p)$ process (X_t) is given by

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p}, \quad (\epsilon_t) \quad WN(\sigma^2).$$

- (i) State without proof the condition for stationarity.
- (ii) Derive the Yule-Walker equations for the autocorrelation (ρ_k) .
- (iii) State the general solution of the Yule-Walker equations.

Q2. The $MA(1)$ process (X_t) is given by

$$X_t = \epsilon_t + \theta \epsilon_{t-1}, \quad |\theta| < 1, \quad (\epsilon_t) \quad WN(\sigma^2).$$

Find

- (i) the variance $\gamma_0 = \text{var} X_t$,
- (ii) the autocovariance $\gamma_k = \text{cov}(X_t, X_{t+k})$,
- (iii) the autocorrelation $\rho_k = \text{corr}(X_t, X_{t+k})$.

Q3. The time-series model is given by

$$X_t = X_{t-1} - \frac{1}{4} X_{t-2} + \epsilon_t + \frac{1}{2} \epsilon_{t-1}, \quad (\epsilon_t) \quad WN(\sigma^2).$$

- (i) Classify (X_t) within the $ARIMA$ class.
- (ii) Show that (X_t) is stationary and invertible.

NHB