smfsoln3(1617).tex

SMF SOLUTIONS 3. 30.10.2017

Q1. One can notice the following effects (cf Lai-Xing):

1. The first component can be interpreted as a *parallel shift component. The factor loadings are roughly constant among maturities, meaning the change in the rate for a maturity is roughly the same for other maturities. Consequently, the first factor accounts for the "average rate".*
2. *The second component corresponds to a tilt . The factor loadings of the second component have a monotonic change with maturities: changes in long-maturity and short maturity have opposite signs. The second factor consequently accounts for the "slope" across maturities.*
3. *The third component is the curvature . The factor loadings of the third component are different for the mid-term rates and the average of long- and short-term rates, revealing a curvature resembling the convex shape of the relationship between the rates and their maturities.*

Q2.

1. The correlation matrix is

$$\begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix},$$

which has eigenvalues 1.8 and 0.2 and eigenvectors $(1,1)$ and $(1,-1)$. So most of the variation is in the direction $(1,1)$. This is expected, given the high correlation.

2. The covariance matrix is

$$\begin{pmatrix} 1,000,000 & 800 \\ 800 & 1 \end{pmatrix}.$$

This has eigenvalues $\approx 1,000,000$ and ≈ 0.36 with eigenvectors (approximately) $(1250,1)$ and $(-0.0008,1)$. The first variable dominates the PCA.

3. The variance of Variable 1 is now 1×10^{-6} and the covariance matrix is
$$\begin{pmatrix} 1 \times 10^{-6} & 8 \times 10^{-4} \\ 8 \times 10^{-4} & 1 \end{pmatrix},$$
which will display the opposite behaviour to (ii) and the second variable will dominate the PCA. The correlation matrix will not change, no matter the scaling of the variables. To use correlation matrices for PCA in R use the command `princomp(yourdata, cor=T)`.

NHB/TLS