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Lecture 0. 12.1.2012.

MA414 STOCHASTIC ANALYSIS

Professor N. H. Bingham

Lectures S421 Thursdays 11-1, Seminar E168 Mon 5-6

B310; 020-7955-6282; n.bingham@lse.ac.uk; office hour Monday 3:30 – 4:30

Recommended texts:

[W] David WILLIAMS, *Probability with martingales*. Cambridge Univ. Press, 1991.

[S] René L. SCHILLING, *Measures, integrals and martingales*. Cambridge Univ. Press, 2005.

[BK] N. H. BINGHAM and Rüdiger KIESEL, *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed. Springer, 2004 (1st ed. 1998).

Books for Reference:

[GS] G. R. GRIMMETT and D. J. STIRZAKER, *Probability with random processes*, 3rd ed. Oxford Univ. Press, 2001.

[O] B. OKSENDAL, *Stochastic differential equations: An introduction with applications*, 6th ed. Springer, 2003.

[P] P. PROTTER, *Stochastic integration and differential equations: A new approach*. Springer, 1990.

[K] O. KALLENBERG, *Foundations of modern probability*, 2nd ed., Springer, 2002.

[D] J. L. DOOB, *Stochastic processes*, Wiley, 1953.

[RY] D. REVUZ and M. YOR, *Continuous martingales and Brownian motion*, 3rd ed., Springer, 1999 (1st ed. 1991, 2nd ed. 1994).

[A, AD] R. B. ASH, *Real analysis and probability*, Academic Press, 1972 (2nd ed., with C. DOLEANS-DADE, *Probability and measure theory*, AP, 2000).

[BF] N. H. BINGHAM and John M. FRY, *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), 2010.

Course outline (20 lectures + 10 classes, 12 January – 15 March + 23 and 30 April 2012)

I. Analysis; Probability. [L1-3: 6 hours]

1. Lebesgue measure and integral [L1]
2. General measure and integral [L1]
3. Probability [L1]
4. Stieltjes integrals [L1]
5. Modes of convergence [L2]
6. Characteristic functions [L2]
7. Independence [L2]
8. Weak Law of Large Numbers; Central Limit Theorem [L2]
9. The Borel-Cantelli lemmas and the zero-one law [L3]
10. Infinite product measures; replication and copies [L3]
11. The Strong Law of Large Numbers (SLLN) [L3]

II. Stochastic processes [L4-8: 9 hours]

1. Conditional expectation [L4]
2. Properties of conditional expectation [L4]
3. Filtrations [L4]
4. Finite-dimensional distributions [L4]
5. Martingales: discrete time [L5]
6. Uniform integrability and martingales [L6]
7. Martingales in continuous time [L6]
8. Brownian motion [L7]
9. Quadratic variation of Brownian motion [L7,8]
10. Properties of Brownian motion [L8]

III. Stochastic (Itô) integration [L8-10: 5 hours]

1. Stochastic integration [L8,9]
2. Itô's lemma [L9]
3. Geometric Brownian motion [L9]
4. Stochastic calculus for Black-Scholes models; Girsanov's theorem [L9, 10]
5. Stochastic differential equations [L10]
6. Semi-martingales [L10]

Examination. Six questions, do 4; 2 hours.

Last year's Exam and Mock Exam (+ Solutions) are on the website, together with all other material from last year. This year will be very similar.

NHB, 12.1.2012

