ma414prob4.tex

MA414 PROBLEMS 4. 2.2.2012

Q1. If X_n are independent, show that the event $A := \{\sum X_n \text{ converges}\}$ has $P(A) = 0$ or 1.

Q2. (i) If $f(x) := x^{-3/2} \exp(-1/(2x))/\sqrt{2\pi}$ on $(0, \infty)$ (Lévy's density), show that the Laplace-Stieltjes transform (LST) of f is

$$\phi(s) := \int_0^\infty e^{-sx} f(x) dx = e^{-\sqrt{2s}} \qquad (s \geq 0).$$

Deduce that f is indeed a probability density function on $(0, \infty)$.
(ii) Show that for X, Y independent non-negative random variables with LSTs $\phi(s)$, $\psi(s)$, $X + Y$ has LST $\phi(s)\psi(s)$: LST of independent sum = product of LSTs.
(iii) Deduce that if $X_1, \ldots, X_n$ are independent and identically distributed to X_1,

$$X_1 + \ldots + X_n =_d X_1/n^2$$

(where $=_d$ denotes equality of distribution).
(iv) The Strong Law of Large Numbers says that, with $X_1, X_2, \ldots$ iid with mean μ,

$$(X_1 + \ldots + X_n)/n \to \mu \qquad (n \to \infty),$$

and then

$$(X_1 + \ldots + X_n)/n^2 \to 0.$$

How is this consistent with (iii)?

NHB