ma414prob7.tex

PROBLEMS 7. 23.2.2012

Q1 *(Conditional Minkowski Inequality)*. Show that for $p \geq 1$, $X, Y \in L_p$, $\mathcal{C}$ a σ-field,

$$(E[|X+Y|^p)|\mathcal{C}])^{1/p} \leq (E[|X|^p)|\mathcal{C}])^{1/p} + (E[|Y|^p)|\mathcal{C}])^{1/p}.$$

Q2. Recall that $U \sim U(0,1)$ corresponds to an infinite sequence of independent coin-tosses $(\epsilon_n)_{n=1}^{\infty}$ under the dyadic expansion $U = \sum_1^{\infty} \epsilon_n/2^n$. By rearranging one sequence into infinitely many, or otherwise, show how from one random variable $U \sim U(0,1)$ we can generate infinitely many independent random variables $U_n \sim U(0,1)$.

Q3.(i) Write Φ for the standard normal distribution function ($\Phi = N(0,1)$), Φ^{-1} for its inverse function (there are no problems defining this, in the usual way, as Φ is continuous and strictly increasing). If $U \sim U(0,1)$, show that $X := \Phi^{-1}(U) \sim N(0,1)$.
(ii) Deduce that from one $U \sim U(0,1)$ we can generate infinitely many independent $N(0,1)$ random variables.
(iii) Deduce that from one $U \sim U(0,1)$ we can generate a Brownian motion.
(iv) Deduce that from one $U \sim U(0,1)$ we can generate infinitely many independent Brownian motions.

Q4. For $B = (B_t)$ BM, show that $(B_t^2 - t)$ is a mg. Give its Doob-Meyer decomposition, and deduce that BM has QV t.

NHB