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STOCHASTIC PROCESSES

Professor N. H. BINGHAM, Spring 2010

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Course website: My homepage, link to Stochastic Processes.

Recommended Texts:
[W] David WILLIAMS, *Probability with martingales.* Cambridge Univ. Press, 1991.
[S] René L. SCHILLING, *Measures, integrals and martingales.* Cambridge Univ. Press, 2005.
[BK] N. H. BINGHAM and Rüdiger KIESEL, *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed. Springer, 2004.
Books for Reference:
[GS] G. R. GRIMMETT and D. J. STIRZAKER, *Probability with random processes*, 3rd ed. Oxford Univ. Press, 2001.
[O] B. OKSENDAL, *Stochastic differential equations: An introduction with applications*, 6th ed. Springer, 2003.
[P] P. PROTTER, *Stochastic integration and differential equations: A new approach.* Springer, 1990.
[K] O. KALLENBERG, *Foundations of modern probability*, 2nd ed., Springer, 2002.
[D] J. L. DOOB, *Stochastic processes*, Wiley, 1953.

Course Outline (30 lectures, 10 weeks, 3 lectures pw)
I. Analysis; Measure theory [9 lectures]
II. Probability; Conditional expectation [7]
III. Martingales; Brownian motion [9]
IV. Stochastic integration: Itô integrals, Iô's formula [5]

The course splits into two parts: Part 1 (Ch. I, II), on measure theory and probability, which is static (time does not play a role), and Part 2 (Ch. III, IV), on stochastic processes and integration, which is dynamic, and time is crucial. Roughly speaking, [S] is good for Part 1, [W] for both parts, but more for Part 1 as it doesn't do continuous time; [BK] is suitable for both, with [S] and [W] a back-up.

A word on the division of time: you have probably heard how vital stochastic integration is to the subject. It is – but most of the hard work is preparation, on integration [Ch. I] and stochastic processes [Ch. III]. Again, you have probably heard how important martingales are [Ch. II]: true, but most of the hard work is preparation on conditional expectation [Ch. II].

A word on hard theorems, proved and omitted. The hard theorems we prove (don't worry – *neither proof is examinable*!) are Kolmogorov's Strong Law of Large Numbers [Ch. II, L14] and Wiener's Theorem on existence of Brownian motion [Ch. III, L20-21]. The hard theorem whose proof we omit is Carathéodory's Extension Theorem (and in particular, existence of Lebesgue measure) [Ch. I, L3-4].

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12. Properties of conditional expectations [L15,16]
13. Filtrations [L16]

III. Martingales; Brownian motion [9]
1. Filtrations; finite-dimensional distributions [L17]
2. Martingales: discrete time [L17,18,19]
3. Martingales: continuous time [L19]
4. Other classes of process (Gaussian, Markov, diffusions) [L20]
5. Brownian motion (BM) [L20-23]
6. Point processes; Poisson processes [L23,24]
7. Lévy processes [L24,25]

IV. Stochastic integration: Itô integrals, Iô's formula [5]
1. Stochastic integrals [L26,27]
2. Itô's Lemma [L27,28]
3. Geometric Brownian motion [L28]
4. Stochastic calculus for Black-Scholes models; Girsanov's Theorem [L28,29]
5. Stochastic differential equations (SDEs) [L29,30]
6. Semi-martingales [L30]

We add some non-examinable background material in a handout. (This is what would be in the text of three more lectures, if we had 11 weeks rather than 10.) We also add a Dramatis Personae, on who did what when (for interest only – not examinable).