

Problems 5. 12.11.2010

Q1. (i) Show that the standard normal distribution $N(0, 1)$ has CF $e^{-t^2/2}$.
(ii) Deduce that the general normal distribution $N(\mu, \sigma)$ has CF $\exp\{i\mu t - \sigma^2 t^2/2\}$.

Q2. (i) Show that the symmetric exponential distribution SE with density

$$f(x) := e^{-|x|}/2$$

has CF

$$\phi(t) = 1/(1 + t^2).$$

(One can do this by Real Analysis – integrate by parts twice.)

(ii) Show that the Cauchy distribution with density

$$f(x) = \frac{1}{\pi(1 + x^2)}$$

has CF

$$\phi(t) = e^{-|t|}.$$

(This uses Complex Analysis, and Jordan's Lemma.)

Q3. Comment on the similarity between density and CF in Q1, and between Q2 (i) and (ii).

NHB