spprob6.tex

Problems 6. 19.11.2010

Q1. Simulating infinitely many copies of BM from one U(0,1).

We showed in L13 that from a simulation of one U(0, 1) we can simulate a sequence of independent copies of N(0, 1), and so a Brownian motion. Extend this to show that we can also simulate a sequence of independent BMs ["as many as we like" would do to handle our needs in simulating financial data].

In what follows, the probability space is $(\Omega, \mathcal{A}, \mathcal{P})$ and \mathcal{B} is a sub- σ -field of \mathcal{A} .

Q2. Conditional Monotone Convergence Theorem. If $X_n \ge 0$, $X_n \uparrow X$ and $X \in L_1$, show that

$$E[X_n|\mathcal{B}] \uparrow E[X|\mathcal{B}].$$

Q3. Conditional Fatou Lemma. If $X_n \ge 0, X_n \in L_1$ and $E[\liminf X_n] < \infty$, show that

 $E[\liminf X_n|\mathcal{B}] \leq \liminf E[X_n|\mathcal{B}].$

Q4. Conditional Dominated Convergence. If $|X_n| \leq Y, Y \in L_1, X_n \to X$, show that

 $E[X_n|\mathcal{B}] \to E[X|\mathcal{B}].$

Q5. If X_1, \ldots, X_n are independent with the symmetric Cauchy distribution, show that $(X_1 + \ldots + X_n)/n$ has the same distribution as X_1 , i.e. symmetric Cauchy. Why does this not contradict the SLLN?

NHB