spprob6.tex

Problems 7. 26.11.2010

Q1. *Conditional Hölder inequality.* For $p, q > 1$, $1/p + 1/q = 1$, $f \in L^p$, $g \in L^q$ (so $fg \in L^1$ by Hölder's inequality) and $\mathcal{B} \subset \mathcal{A}$, show that

$$|E[fg|\mathcal{B}]| \leq (E[|f|^p|\mathcal{B})^{1/p}.(E[|g|^q|\mathcal{B})^{1/q}.$$

Q2. *Conditional Minkowski inequality.* For $p \geq 1$, $f, g \in L^p$, $\mathcal{B} \subset \mathcal{A}$, show that

$$(E[|f|^p|\mathcal{B})^{1/p} + (E[|g|^p|\mathcal{B})^{1/p} \leq (E[|f+g|^p|\mathcal{B})^{1/p}.$$

Q3. *Conditional Jensen inequality.* If ϕ is convex, X and $\phi(X)$ are both integrable and $\mathcal{B} \subset \mathcal{A}$, show that

$$\phi(E[X|\mathcal{B}]) \leq E[\phi(X)|\mathcal{B}]$$

(you may quote that a convex function is continuous).

Q4. *Scheffé's Lemma.* If f_n, f are probability densities, and $f_n \to f$ a.e., show that (for Borel sets $B \in \mathcal{B}$)

$$\sup_{B \in \mathcal{B}} |\int_B f_n - \int_B f| \leq \int |f_n - f| \to 0.$$

NHB