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STOCHASTIC PROCESSES

Professor N. H. BINGHAM, Autumn 2011

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Course website: My homepage, link to Stochastic Processes.

Recommended Texts:

[W] David WILLIAMS, Probability with martingales. Cambridge Univ. Press, 1991.

[S] René L. SCHILLING, Measures, integrals and martingales. Cambridge Univ. Press, 2005.

[BK] N. H. BINGHAM and Rüdiger KIESEL, *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed. Springer, 2004. Books for Reference:

[GS] G. R. GRIMMETT and D. J. STIRZAKER, *Probability with random processes*, 3rd ed. Oxford Univ. Press, 2001.

[O] B. OKSENDAL, Stochastic differential equations: An introduction with applications, 6th ed. Springer, 2003.

[P] P. PROTTER, Stochastic integration and differential equations: A new approach. Springer, 1990.

[K] O. KALLENBERG, Foundations of modern probability, 2nd ed., Springer, 2002.

[D] J. L. DOOB, Stochastic processes, Wiley, 1953.

Course Outline (30 lectures, 10 weeks, 3 lectures pw)

I. Analysis; Measure theory [9 lectures]

II. Probability; Conditional expectation [7]

III. Martingales; Brownian motion [9]

IV. Stochastic integration: Itô integrals, Itô's formula [5]

The course splits into two parts: Part 1 (Ch. I, II), on measure theory and probability, which is static (time does not play a role), and Part 2 (Ch. III, IV), on stochastic processes and integration, which is dynamic, and time is crucial. Roughly speaking, [S] is good for Part 1, [W] for both parts, but more for Part 1 as it doesn't do continuous time; [BK] is suitable for both, with [S] and [W] a back-up.

A word on the division of time: you have probably heard how vital stochastic integration is to the subject. It is – but most of the hard work is preparation, on integration [Ch. I] and stochastic processes [Ch. III]. Again, you have probably heard how important martingales are [Ch. II]: true, but most of the hard work is preparation on conditional expectation [Ch. II].

A word on hard theorems, proved and omitted. The hard theorems we prove (don't worry – proofs not examinable!) are Kolmogorov's Strong Law of Large Numbers [Ch. II, L14] and Wiener's Theorem on existence of Brownian motion [Ch. III, L20-21]. The hard theorem whose proof we omit is Carathéodory's Extension Theorem (and in particular, existence of Lebesgue measure) [Ch. I, L3-4].

Table of Contents

- I. Analysis; Measure theory [9 lectures]
- 1. Length, area and volume [L1]
- 2. Classes of sets [L2]
- 3. Measures [L3,4]
- 4. The Lebesgue integral [L5,6]
- 5. Properties of the integral [L6,7,8]
- 6. Stieltjes integrals [L8]
- 7. Further results [L9]

II. Probability; Conditional expectation [7]

- 1. Probability spaces [L10]
- 2. Random variables [L10]
- 3. Expectations [L10]
- 4. Modes of convergence [L11]
- 5. Characteristic functions [L11]
- 6. Independence [L11,12]
- 7. Weak Law of Large Numbers and Central Limit Theorem [L12]
- 8. The Borel-Cantelli Lemmas and the Zero-One Law [L13]
- 9. Infinite product measures; replication and copies [L13,14]
- 10. Strong Law of Large Numbers [[L14]
- 11. Conditional expectations [L15]

- 12. Properties of conditional expectations [L15,16]
- 13. Filtrations [L16]

III. Martingales; Brownian motion [9]

- 1. Filtrations; finite-dimensional distributions [L17]
- 2. Martingales: discrete time [L17]
- 3. Martingale convergence and uniform integrability [L18-19]
- 4. Martingales: continuous time [L19]
- 5. Other classes of process (Gaussian, Markov, diffusions) [L20]
- 6. Brownian motion (BM) [L20-23]
- 7. Point processes; Poisson processes [L23,24]
- 8. Lévy processes [L24,25]

IV. Stochastic integration: Itô integrals, Itô's formula [5]

- 1. Stochastic integrals [L26,27]
- 2. Itô's Lemma [L27,28]
- 3. Geometric Brownian motion [L28]
- 4. Stochastic calculus for Black-Scholes models; Girsanov's Theorem [L28,29]
- 5. Stochastic differential equations (SDEs) [L29,30]
- 6. Semi-martingales [L30]

We add some non-examinable background material in a handout. (This is what would be in the text of three more lectures, if we had 11 weeks rather than 10.) We also add a Dramatis Personae, on who did what when (for interest only – not examinable).