spprob10.tex

Problems 10. 17.12.2010

Q1. (i) For N Poisson distributed with parameter λ and $X_1, X_2, \ldots$ independent of each other and of N, each with distribution F with mean μ, variance σ^2 and characteristic function $\phi(t)$, show that the compound Poisson distribution of

$$Y := X_1 + \ldots + X_N$$

has characteristic function $\psi(t) = \exp\{-\lambda(1 - \phi(t))\}$, mean $\lambda\mu$ and variance $\lambda E[X^2]$.
(ii) Obtain the mean and variance of Y also from the Conditional Mean Formula and the Conditional Variance Formula.

Q2. For $B = (B_t)$ Brownian motion and $M = (M_t)$, where

$$M_t := (B_t^2 - t)^2 - 4\int_0^t B_s^2 ds,$$

(i) find the stochastic differential of M. Hence or otherwise, express M as an Itô integral, and show that M is a continuous martingale starting at 0.
(ii) Find the quadratic variation $[M]_t$ of M_t.

Q3. For $V = (V_t)$ the solution to the Ornstein-Uhlenbeck SDE (OU):
(i) By using the Itô isometry, or otherwise, show that V_t has distribution $N(0, \sigma^2(1 - e^{-2\beta t})/(2\beta))$.
(ii) By (i) and independence of Brownian increments, or otherwise, show that the covariance is

$$cov(V_t, V_{t+u}) = \sigma^2 e^{-2\beta u}(1 - 2e^{-2\beta t}) \qquad (u \geq 0).$$

(iii) Show that V is Gaussian and Markov.

NHB