spprob6.tex

Problems 6. 18.11.2010

Q1. *Simulating infinitely many copies of BM from one U(0,1).*

We showed in L13 that from a simulation of one $U(0,1)$ we can simulate a sequence of independent copies of $N(0,1)$, and so a Brownian motion. Extend this to show that we can also simulate a sequence of independent BMs ["as many as we like" would do to handle our needs in simulating financial data].

In what follows, the probability space is $(\Omega, \mathcal{A}, \mathcal{P})$ and $\mathcal{B}$ is a sub-σ-field of $\mathcal{A}$.

Q2. *Conditional Monotone Convergence Theorem.*

If $X_n \geq 0$, $X_n \uparrow X$ and $X \in L_1$, show that

$$E[X_n|\mathcal{B}] \uparrow E[X|\mathcal{B}].$$

Q3. *Conditional Fatou Lemma.*

If $X_n \geq 0$, $X_n \in L_1$ and $E[\liminf X_n] < \infty$, show that

$$E[\liminf X_n|\mathcal{B}] \leq \liminf E[X_n|\mathcal{B}].$$

Q4. *Conditional Dominated Convergence.*

If $|X_n| \leq Y$, $Y \in L_1$, $X_n \to X$, show that

$$E[X_n|\mathcal{B}] \to E[X|\mathcal{B}].$$

Q5. If $X_1, \ldots, X_n$ are independent with the symmetric Cauchy distribution, show that $(X_1 + \ldots + X_n)/n$ has the same distribution as X_1, i.e. symmetric Cauchy. Why does this not contradict the SLLN?

NHB