spprob7.tex

Problems 7. 25.11.2011

Q1. Conditional Hölder inequality. For p, q > 1, 1/p + 1/q = 1, $f \in L^p$, $g \in L^q$ (so $fg \in L^1$ by Hölder's inequality) and $\mathcal{B} \subset \mathcal{A}$, show that

$$|E[fg|\mathcal{B}]| \le (E[|f|^p|\mathcal{B})^{1/p} \cdot (E[|g|^q|\mathcal{B})^{1/q}.$$

Q2. Conditional Minkowski inequality. For $p \ge 1$, $f, g \in L^p$, $\mathcal{B} \subset \mathcal{A}$, show that

$$(E[|f|^{p}|\mathcal{B})^{1/p} + (E[|g|^{p}|\mathcal{B})^{1/p} \le (E[|f+g|^{p}|\mathcal{B})^{1/p}.$$

Q3. Conditional Jensen inequality. If ϕ is convex, X and $\phi(X)$ are both integrable and $\mathcal{B} \subset \mathcal{A}$, show that

$$\phi(E[X|\mathcal{B}]) \le E[\phi(X)|\mathcal{B}]$$

(you may quote that a convex function is continuous).

Q4. Scheffé's Lemma. If f_n , f are probability densities, and $f_n \to f$ a.e., show that (for Borel sets $B \in \mathcal{B}$)

$$\sup_{B \in \mathcal{B}} \left| \int_{B} f_{n} - \int_{B} f \right| \leq \int \left| f_{n} - f \right| \to 0.$$
 NHB