

## MATL481: INTEREST RATE THEORY

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*Course website* My homepage, as for MATL480 last term.

### *Books*

#### *Course text*

The standard work is co-authored by my Imperial College colleague Professor Damiano Brigo:

[BM] Damiano BRIGO & Fabio MERCURIO, *Interest rate models – Theory and practice, with Smile, inflation and credit*, 2nd ed., Springer, 2006, 981p (1st ed. 2001)

(pre-Crash). We use this, together with three later works by Brigo (et al.) (post-Crash):

[BPT] Damiano BRIGO, Andrea PALLAVICINI & Roberto TORRESETTI, *Credit models and the Crisis: A journey into CDOs, copulas, correlations and dynamic models*, Wiley, 2010;

[BMP] Damiano BRIGO, Massimo MORINI & Andrea PALLAVICINI, *Counterparty credit risk, collateral and funding, With pricing cases for all asset classes*, Wiley, 2013;

[B] Damiano BRIGO, *Counterparty risk, collateral and funding across asset classes with arbitrage-free dynamic models*. London Graduate School in Mathematical Finance - MF6 Course, Nov 6, 7, 13, 14, 2012 (Internet).

Also useful:

[Dav] M. H. A. DAVIS, *Mathematical finance: A very short introduction*, OUP, 2019.

[R1] Riccardo REBONATO, *Interest-rate option models: Understanding*,

*analysing and using models for exotic interest-rate options*, Wiley, 1998.  
[R2] Riccardo REBONATO, *Volatility and correlation in the pricing of equity, FX and interest-rate options*. Wiley, 1999.  
[JW] Jessica JAMES & Nick WEBBER, *Interest rate modelling*, Wiley, 2000,  
[Z] Rudi ZAGST, *Interest rate management*, Springer, 2002,  
[C] Andrew J. G. CAIRNS, *Interest rate models: An introduction*, Princeton University Press, 2004.

In addition, most books on mathematical finance have at least a chapter on interest rates; see e.g.

[BK] N. H. BINGHAM & Rüdiger KIESEL, *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., Springer, 2004 (1st ed. 1998),  
[DJ] R.-A. DANA & M. JEANBLANC, *Financial market in continuous time*, Springer, 2003,  
[S1] Steven E. SHREVE, *Stochastic calculus for finance, I: The binomial asset pricing model*, Springer, 2004,  
[S2] Steven E. SHREVE, *Stochastic calculus for finance, II: Continuous-time models*, Springer, 2004.

#### *Financial Statistics.*

[LX] Tze Leung LAI and Hipeng XING, *Statistical models and methods for financial markets*, Springer, 2008.  
[MFE] A. J. McNEIL, Rüdiger FREY and Paul EMBRECHTS, *Quantitative risk management: Concepts, tools, techniques*. Princeton UP, 2005.  
[G] J. GATHERAL, *The volatility surface: A practitioner's guide*. Wiley, 2006.

#### *Credit risk*

In addition to [BBT] and [BMP] above, see also  
[BR] Tomasz BIELECKI and Marek RUTKOWSKI, *Credit risk: Modeling, valuation and hedging*. Springer, 2002.  
[L] David LANDO, *Credit risk modelling*, Princeton University Press, 2004.

#### *General interest*

[Lew] Michael LEWIS: *Liar's poker*. Hodder & Stoughton, 1989, 2016.  
[Sor] Andrew Ross SORKIN, *Too big to fail: Inside the battle to save Wall Street*. Penguin, 2009.  
[Cha] Ha-Joon CHANG, *23 things they don't tell you about capitalism*. Penguin, 2010.

[VF1] Liam VAUGHAN and Gavin FRENCH, *The Fix: How bankers lied, cheated and colluded to rig the world's most important number*, Wiley, 27 January 2017.

The course splits into two parts: the minor one, Part 1 (3 weeks, Ch. I-IV) on the older material, and the major one, Part 2 (4 weeks, Ch. V, VI) on the more recent material: *market models* (V), which date from 1997, and *credit risk* (VI), already the subject of Lando's 2004 book above, but made much more important by the Crash of 2007/8. It now permeates the whole subject, including market models.

## CONTENTS

### Part 1 [w1-3]

#### I. BACKGROUND [1w; 1ab]

1. Revision of the Black-Scholes formula and PDE; Notes [1a]
2.  $\mathbb{P}$ -measure,  $\mathbb{Q}$ -measure and pricing kernels [1a]
3. “Big-picture stuff”: MATL480 and MATL481, and beyond [1a,b]
4. Historical background [1b]
5. Assumptions [1b]
6. Are interest-rates positive? [1b]
7. Econometrics; macroeconomic policy [1b]

#### II. INTEREST-RATE PRODUCTS AND DERIVATIVES [ $\frac{1}{2}$ w; 2a]

1. Terminology; zero-coupon bonds (ZCB); LIBOR
2. Quantitative easing (QE)
3. Products not depending on the curve dynamics: forward-rate agreements (FRA), interest-rate swaps (IRS)
4. Products depending on the curve dynamics: caps/caplets, floors/floorlets, swaptions

#### III. SPOT-RATE MODELS [1w; 2b, 3a]

1. Possible model choices [2b]
2. Vasicek model, 1977 [2b]
3. Cox-Ingersoll-Ross (CIR) model, 1985 [2b]
4. Affine term-structure models (ATM) [2b]
5. Exponential Vasicek model [2b]

6. Vasicek model (continued): Objective measure; econometrics, statistics, historical estimation [2b]
7. Spot rate: Choice of model [2b, 3a]
8. Multidimensional models: How many factors? [3a]

#### **IV. FORWARD-RATE MODELS** [ $\frac{1}{2}$ w; 3a,b]

1. Heath-Jarrow-Morton (HJM) model, 1992 [3a]
2. Multidimensional models and correlations [3a]
3. G2++: Gaussian two-factor additive models [3b]
4. What do we measure? What should we measure? [3b]

### **Part 2** [w4-7]

#### **V. MARKET MODELS** [ $2\frac{1}{2}$ w; 4ab, 5ab, 6a]

1. Introduction [4a]
2. Black's caplet formula [4a]
3. LIBOR market models (LMM) [4a]
4. Swap market models (SMM); Black's swaption formula [4a]
5. The change-of-numeraire formula [4a,b]
6. LMM dynamics [4b]
7. The Heath-Jarrow-Morton (HJM) drift condition [4b]
8. LMM: calibration to market data [5a]
9. Instantaneous correlations: parametric forms [5a]
10. Monte-Carlo pricing of swaptions with LMM [5a,b]
11. Analytical pricing of swaptions with LMM [5b]
12. Instantaneous correlations as inputs: The historical matrix [5b]
13. Smile: volatility modelling. Term structure of volatility; Breeden-Litzenberger and Dupire formulae [5b,6a]
14. Is volatility rough? Fractional Brownian motion (fBM) [6a]
15. Other models: [6a]
  1. SABR; 2. Flesaker-Hughston; 3. Rogers; 4. Brody-Hughston

#### **VI. CREDIT RISK** [ $1\frac{1}{2}$ w; 6b, 7ab]

1. Introduction [6b]
2. Credit default swaps (CDS) [6b]
3. Intensity (reduced-form) models; Lando's formula [6b,7a]
4. Firm value (structural) models [7a]
5. Dependence and copulas: Tails and diversification [7a,b]

6. Corporate bonds; credit rating; credit scoring; toxic debt [7b]
7. Concluding remarks [7b]; Postscript [7b]

## Abbreviations

There are unfortunately rather a lot of new terms to learn, many long enough for an abbreviation to be useful (no one likes ‘alphabet soup’, but imagine having to spell out BBC, NHS, RAF, hcf. lcm etc. all the time!).

ABS: Asset-backed securities (VI.5)  
 BDT: Black-Derman-Toy (model)  
 BGM: Brace-Gatarek-Musiela (market) (model)  
 CDO: Collateralised debt obligation (VI.5)  
 CDS: Credit default swap (VI.2)  
 CEV: Constant elasticity of volatility (V.13)  
 CIR: Cox-Ingersoll-Ross (model) (III.3)  
 CMS: Constant maturity swap  
 FRA: Forward-rate agreement (II.2)  
 GPC: General piecewise-constant (V.8)  
 HJM: Heath-Jarrow-Morton (HJM) model (IV.1, V.7)  
 IRS: Interest-rate swap (II.2)  
 ISDA: International Swaps and Derivatives Association (VI.2)  
 LGD: Loss given default (VI.2)  
 LIBOR: London Inter-Bank Offer Rate (I.4)  
 LMM: LIBOR market model (V.3,6,8,10,11)  
 LV: Local volatility (V.13)  
 MBS: Mortgage-backed securities (VI.5)  
 OIS: Overnight indexed swaps (I.4)  
 PCA: Principal components analysis (V.4)  
 PLE: Parametric linear-exponential (V.8)  
 REC: Recovery (VI.2)  
 SEC: Securities & Exchange Commission (VI.5)  
 SMM: Swap market model (V.4)  
 SONIA Sterling Overnight Index Average (I.4)  
 SPC: Separable piecewise-constant (V.8)  
 SV: Stochastic volatility (V.13)  
 TSOV: Term structure of volatilities (V.8)  
 ZCB: Zero-coupon bond (II.1)