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Part 1

I. BACKGROUND

1. Revision of the Black-Scholes formula and PDE

We recall the Black-Scholes model, with one risky asset: we have a *bank account* $B = (B_t)$, and a *risky asset* $S = (S_t)$, with dynamics

$$dB_t = rB_t dt$$

(so r is the riskless interest rate, or *spot rate*, or *short rate*; see Ch. III below),

$$dS_t = S_t(\mu dt + \sigma dW_t) \qquad (GBM)$$

('W for Wiener', as we are using 'B for bank', rather than 'B for Brownian'), the stochastic differential equation (SDE) for geometric Brownian motion (GBM). Here μ is the mean return rate on the stock, σ is the volatility of the stock, $W = (W_t)$ is the driving noise – Brownian motion (BM), representing the uncertainty or unpredictability in this uncertain and unpredictable world. So our holding at time t is described by a bivariate stochastic process (B_t, S_t), on a filtered probability space (or stochastic basis) $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$. A *trading strategy* is a pair of stochastic processes $\phi = (\phi^B, \phi^S)$; here ϕ_t^B, ϕ_t^S are the amounts of cash and stock held at time t. Both processes are *predictable*: the value at time t will be known immediately before t. Left-continuity suffices for this, and this always holds in the Black-Scholes model, where everything is continuous. The *value process* is the process V obtained by following strategy ϕ:

$$V_t(\phi) = \phi_t^B B_t + \phi_t^S S_t; \qquad (V)$$

the first term is the cash part, the second the risky-asset part. The *gain process* is

$$G_t(\phi) := \int_0^t \phi_u^B dB_u + \int_0^t \phi_u^S dS_u, \qquad (G)$$

the net gain (profit or loss) from following strategy ϕ. The strategy is *self-financing* (SF) if the change in G is due only to changes in B and S (so that the trader can trade with no need for extra funds from his firm, and no profit diverted for his/his firm's use):

$$dV_t(\phi) = dG_t(\phi),$$

i.e.

$$dV_t = d(\phi_t^B B_t + \phi_t^S S_t) = \phi_t^B dB_t + \phi_t^S dS_t. \tag{SF}$$

A *contingent claim* Y is just an $\mathcal{F}_T$-measurable random variable (here T is the *expiry time*; think of Y as the payoff (to the holder) or claim (to the writer) of an option expiring at time T; this is contingent (= dependent) on what happens – whether the option expires in/at/out of the money (ITM/ATM/OTM), etc. A (contingent) claim is *attainable* if there exists a SF strategy ϕ attaining it, i.e. such that

$$V_T(\phi) = Y.$$

Then ϕ *generates* Y, and $V_t(\phi)$ is the *price* of Y at time t.

Derivation of the Black-Scholes PDE

For an attainable claim on the stock S, its value V_t at time t depends in the stock price S_t,

$$V_t = V(t, S_t).$$

For V suitably smooth – $V \in C^{1,2}([0,t) \times \mathbb{R}^+)$ – Itô's lemma gives

$$dV(t, S_t) = \Big(\frac{\partial V}{\partial t}(t, S_t) + \frac{\partial V}{\partial S}(t, S_t)\mu S_t + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}(t, S_t)\sigma^2 S_t^2\Big)dt + \frac{\partial V}{\partial S}(t, S_t)\sigma S_t dW_t \tag{Ito}$$

(as in MATL480 Ch. V: this uses (GBM) and $dW_t^2 = dt$). For $t \in [0, T]$, define

$$\phi_t^S := \frac{\partial V}{\partial S}(t, S_t), \qquad \phi_t^B := (V_t - \phi_t^S S_t)/B_t;$$

then (V) holds, so the value of this strategy is V. If ϕ is SF (and we will only need to consider SF strategies), (SF) gives, substituting the above,

$$\begin{aligned} dV_t &= \phi_t^B dB_t + \phi_t^S dS_t \\ &= (V(t,S) - \frac{\partial V}{\partial S}(t, S_t))rdt + \frac{\partial V}{\partial S}(t, S_t)S_t)\mu dt + \sigma dW_t). \end{aligned}$$

This and (SF) give us two expressions for dV_t. The coefficients of dW_t are the same; equating the coefficients of dt gives

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S}\mu S_t + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S_t^2 = rV - r\frac{\partial V}{\partial S} + \frac{\partial V}{\partial S}\mu S_t.$$

This gives the famous *Black-Scholes PDE*:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV. \qquad (BS - PDE)$$

We solved this in MATL480 VI.3, for the two cases of payoff (terminal condition at $t = T$)

$$V(T, S) = (S - K)_+ \qquad (BC)$$

(European call option with strike K), giving

$$V(t, S) = S\Phi(d_+) - e^{-r(T-t)} K\Phi(d_-), \quad d_\pm := [\log(S/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{T-t}, \qquad (C)$$

and the corresponding formula (P) for European put options with strike K, where instead

$$V(T, S) = (K - S)_+.$$

Again as in MATL480 VI, the Feynman-Kac theorem allows one to express the solution of the PDE + BC for $V = V(t, x)$,

$$\frac{\partial V}{\partial t} + b(x)\frac{\partial V}{\partial x} + \frac{1}{2}\sigma(x)^2\frac{\partial^2 V}{\partial x^2} = rV, \qquad V(T, x) = f(x) \qquad (PDE)$$

as

$$V(t, x) = e^{-r(T-t)} E^{\mathbb{Q}}_{t,x}[f(X_T)|\mathcal{F}_t], \qquad (RNVF)$$

where under the probability measure $\mathbb{Q}$ the diffusion process X has dynamics, starting from $X = x$ at time t,

$$dX_s = b(X_s)ds + \sigma(X_s)dW^{\mathbb{Q}}_s, \quad s \geq t, \quad X_t = x$$

(here $W^{\mathbb{Q}}$ is standard BM under $\mathbb{Q}$). Specialising to

$$b(x) = rx, \qquad \sigma(x) = \sigma x$$

(so the general PDE becomes the BS PDE), this gives: the unique no-arbitrage price of the claim $Y = (S_T - K)_+$ (European call option) at time $t \in [0, T]$ is

$$V_{BS}(t) = E^{\mathbb{Q}}[e^{-r(T-t)}Y|\mathcal{F}_t];$$

here $\mathbb{Q}$ is the *equivalent martingale measure* (EMM) – the probability measure $\mathbb{Q} \sim \mathbb{P}$ under which the risky-asset price $S_t/B_t = e^{-rt}S_t$ has $\mathbb{Q}$-dynamics

$$dS_t = S_t[rdt + \sigma dW_t^Q].$$

Girsanov's theorem

As we covered this in MATL480, we can be informal here. On a stochastic basis $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$, consider an SDE (under the measure $\mathbb{P}$)

$$dX_t = b(X_t)dt + v(X_t)dW_t, \qquad X_0 = x_0, \qquad \mathbb{P}.$$

Under the relevant technical conditions, define a measure $\mathbb{Q}$ by its Radon-Nikodym derivative w.r.t. $\mathbb{P}$ via

$$\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_t} = \exp\{-\frac{1}{2}\int_0^t \Big(\frac{b^{\mathbb{Q}}(X_s) - b(X_s)}{v(X_s)}\Big)^2 ds + \int_0^t \Big(\frac{b^{\mathbb{Q}}(X_s) - b(X_s)}{v(X_s)}\Big) dW_s\}.$$

Then under $\mathbb{Q} \sim \mathbb{P}$,

$$dW_t^{\mathbb{Q}} = -\Big(\frac{b^{\mathbb{Q}}(X_t) - b(X_t)}{v(X_t)}\Big)dt + dW_t$$

is a BM, and on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$ X satisfies the SDE

$$dX_t = b^{\mathbb{Q}}(X_t)dt + v(X_t)dW_t^{\mathbb{Q}}, \quad X_0 - x_0, \qquad \mathbb{Q}.$$

Note that it is only the *drift* that changes (b becomes $b^{\mathbb{Q}}$). The *diffusion coefficient* v is the same. Indeed, if the diffusion coefficients were different, the two probability measures would not be equivalent. Statistically, what this means is that, given two diffusions with the same diffusion coefficient, we can test for whether their drifts are the same by using a likelihood-ratio test, the test statistic being obtained from the RN derivative above.

So: if we use Girsanov's theorem to move from

$$dS_t = S_t(\mu dt + \sigma dW_t) \qquad (GBM - \mathbb{P})$$

to

$$dS_t = S_t(rdt + \sigma dW_t), \qquad (GBM - \mathbb{Q})$$

the RN derivative is

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\{-\frac{1}{2}\Big(\frac{\mu - r}{\sigma}\Big)^2 T - \Big(\frac{\mu - r}{\sigma}\Big)W_T\}.$$

Recall also from MATL480:

No-Arbitrage Theorem. The market has no arbitrage (is NA) iff EMMs *exist*.

Completeness Theorem. An NA market is complete (i.e. all contingent claims can be replicated) iff EMMs are *unique*.

In reality, **markets are never complete**.

Roughly speaking a market is complete if there are as many assets as independent sources of randomness. But: while there are lots of assets, the uncertain world is so complicated that it contains even more sources of randomness. So: we have to live with incompleteness, non-unique prices (just like out on the High Street!), and bid-ask *spreads*.

The SF strategy ϕ enables the option seller to find a *perfect hedge*, which will cover him against any claim the option buyer may make. This is called *delta-hedging*. For, recall 'the Greeks', the most important and basic of which is Delta, $\Delta := \partial V/\partial S$ (below). However, the option seller only sells the option in the hope of making money by doing so. If he hedges perfectly, he will make no loss – but he will *make no profit* either. So he might as well not bother selling the option in the first place. In practice, he will normally use *partial hedging* – lay off some of the risk, but not all, so as not to lose all potential profit. 'Nothing venture, nothing win'!

Three Greeks.

The Black-Scholes PDE involves three derivative terms (first and second space derivatives, and first time derivative). There is a corresponding link between the three corresponding 'Greeks'. Recall:
Delta,

$$\Delta := \partial V/\partial S;$$

Gamma,

$$\Gamma := \partial \Delta/\partial S = \partial^2 V/\partial S^2;$$

Theta, which measures sensitivity to time:

$$\Theta_t := \partial V_t/\partial t = -\partial V_t/\partial (T-t)$$

(in terms of the remaining time $\tau := T - t$ to expiry). These are linked. For, recall (Ito) (Itô in text, but 'Ito in TeX'):

$$dV(t,S_t) = \frac{\partial V}{\partial t}(t,S_t)dt + \frac{\partial V}{\partial S}(t,S_t)dS_t + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}(t,S_t)\sigma^2 S_t^2 dt.$$

One can see from this equation that it will imply a link between Delta, Gamma and Theta.

To derive this, recall $dS = S(\mu dt + \sigma dW)$, so $(dS)^2 = \sigma^2 S^2 (dW)^2 = \sigma^2 S^2 dt$. This can be written

$$dV = \Theta dt + \frac{1}{2}\Gamma\sigma^2 S^2 dt + \Delta dS.$$

But from the self-financing condition (SF), we also have (where $B = B_t$ is the bank account, so $dB_t = r_t B_t dt$)

$$dV = [(V - \Delta S)/B]dB = (V - \Delta S)rdt + \Delta dS.$$

Equating, the three Greeks are linked to the spot rate by $(dB_t = r_t B_t dt)$

$$r_t V(t,S_t) = r_t \Delta_t S_t + \Theta_t + \frac{1}{2}\Gamma_t \sigma^2 S_t^2.$$

Ornstein-Uhlenbeck (OU)/Vasicek (Vas) process.

(i) The OU SDE $dV = -\kappa V dt + \sigma dW$ (OU) models the velocity of a diffusing particle. The $-\kappa V dt$ term is *frictional drag*, and κ is the *inverse relaxation time*; the σdW term is *noise*, and σ is the *volatility*.

(ii) $e^{-\kappa t}$ solves the corresponding homogeneous DE $dV = -\kappa V dt$. So by *variation of parameters*, take a trial solution $V = Ce^{-\kappa t}$. Then

$$dV = -\kappa C e^{-\kappa t}dt + e^{-\kappa t}dC = -\kappa V dt + e^{-\kappa t}dC,$$

so V is a solution of (OU) if $e^{-\kappa t}dC = \sigma dW$, $dC = \sigma e^{\kappa t}dW$, $C = c + \sigma\int_0^t e^{\kappa u}dW$. So with initial velocity v_0, $V = e^{-\kappa t}C$ is

$$V = v_0 e^{-\kappa t} + \sigma e^{-\kappa t}\int_0^t e^{\kappa u}dW_u.$$

(iii) V comes from W, Gaussian, by linear operations, so is Gaussian.
V_t has mean $v_0 e^{-\kappa t}$, as $E[e^{\kappa u}dW_u] = \int_0^t e^{\kappa u}E[dW_u] = 0$.
By the Itô isometry, V_t has variance

$$E[(\sigma e^{-\kappa t}\int_0^t e^{\kappa u}dW_u)^2] = \sigma^2 e^{-2\kappa t}\int_0^t (e^{\kappa u})^2 du$$

$$= \sigma^2 e^{-2\kappa t}[e^{2\kappa t} - 1]/(2\kappa) = \sigma^2[1 - e^{-2\kappa t}]/(2\kappa).$$

So V_t has distribution $N(v_0 e^{-\kappa t}, \sigma^2(1 - e^{-2\kappa t})/(2\kappa))$.
(iv) For $u \geq 0$, the covariance is $cov(V_t, V_{t+u})$, which is

$$\sigma^2 E[e^{-\kappa t} \int_0^t e^{\kappa v} dW_v . e^{-\kappa(t+u)} (\int_0^t + \int_t^{t+u}) e^{\kappa w} dW_w].$$

By independence of Brownian increments, $\int_t^{t+u}$ contributes 0, so by above

$$cov(V_t, V_{t+u}) = e^{-\kappa u} var(V_t) = \sigma^2 e^{-\kappa u}[1 - e^{-2\kappa t}]/(2\kappa) \to \sigma^2 e^{-\kappa u}/(2\kappa) \quad (t \to \infty).$$

(v) V is Markov (a diffusion), being the solution of the SDE (OU). The limit distribution as $t \to \infty$ is $N(0, \sigma^2/(2\kappa))$ (the *Maxwell-Boltzmann distribution* of Statistical Mechanics). As only the time-difference u survives the passage to the limit $t \to \infty$, the limit process is stationary; it is also Gaussian, and Markov, by above.
(vi) The process shows *mean reversion* – a strong push towards the central value. This is characteristic of interest rates (under normal conditions – post-Crash, interest rates have been stuck at just above zero – unprecedented). The financial relevance is to the *Vasicek model* of interest-rate theory.
(vii) The Vasicek model is widely used because it is analytically tractable, and easy to interpret. Its main drawbacks both stem from its Gaussianity (as do its main advantages!):
(a) negative interest rates;
(b) poor fit to market data: tails too thin, symmetric rather than skew, etc.
In addition:
(c) One-factor models are not capable of capturing all relevant aspects; one needs at least a two- (or three-) factor model, and the Vasicek model does indeed extend easily to higher factors.

2. **$\mathbb{P}$-measure**, **$\mathbb{Q}$-measure and pricing kernels**

Recall (MATL480, Ch. II) that Radon-Nikodym derivatives obey the same rules as derivatives in ordinary calculus: for $\mathbb{P} \sim \mathbb{Q}$,

$$\frac{d\mathbb{P}}{d\mathbb{Q}} d\mathbb{Q} = d\mathbb{P}; \qquad \frac{d\mathbb{Q}}{d\mathbb{P}} d\mathbb{P} = d\mathbb{Q}.$$

So in the RNVF, where we have $E^{\mathbb{Q}}[.]$, which is $\int_{\Omega}[.]d\mathbb{Q}$, we can replace this by $\int_{\Omega}[.].d\mathbb{Q}/d\mathbb{P}.d\mathbb{P}$. So if we write

$$\zeta := \frac{d\mathbb{P}}{d\mathbb{Q}}, \qquad \zeta^{-1} := \frac{d\mathbb{Q}}{d\mathbb{P}},$$

the *pricing kernel*, we can write the RNVF as

$$V(t,x) = e^{-r(T-t)} E_{t,x}[f(X_T)\zeta|\mathcal{F}_t], \qquad (RNVF - \mathbb{P})$$

as a $\mathbb{P}$-expectation, under E, rather than a $\mathbb{Q}$-expectation, under $E^{\mathbb{Q}}$, at the price of introducing an extra factor ζ into the integrand. It is often convenient to do this.
Note. The 'pricing' in the name is evident (risk-neutral *valuation*). For 'kernel': this derives from the subject of *integral equations* (rather like differential equations, but with integrals rather than derivatives), where one typically encounters equations such as

$$\int f(y)k(x,y)dy = g(x),$$

to be solved for the unknown function f with g and k given; here k is called the *kernel*.

Notation.

Because nearly all our expectations here in MATL481 will be under $\mathbb{Q}$, it is convenient to drop the $\mathbb{Q}$ in $E^{\mathbb{Q}}[.]$, $E_t^{\mathbb{Q}}[.]$ and just write $E[.]$, $E_t[.]$. Brownian motion (BM) under $\mathbb{Q}$ will be written $W = (W_t)$.

It is then convenient to recognise the primacy of $\mathbb{Q}$ over $\mathbb{P}$, and replace $\mathbb{P}$ in our notation by $\mathbb{Q}^0$. Then $\mathbb{P}$-expectation and BM will be written

$$E^0[.], \qquad W_0$$

(compare $\mathbb{P}$, $\mathbb{P}^*$ in MATL480). The superscript 0 in E^0 (subscript in W_0) comes from *measures of location* in Statistics, related to choice of *origin* 0 on the line (super not sub for E, as we write $E_t[.]$ for $E[.|\mathcal{F}_t]$ later). This change of origin corresponds to the change of *drift* in Girsanov's theorem.

Terminology.

We call $\mathbb{Q}$ the *risk-neutral* measure, $\mathbb{P} = \mathbb{Q}^0$ the *real-world*, *objective* or *physical* measure. Each is useful, but for different purposes; we return to the interplay between these two aspects in Ch. III.

$\mathbb{P}$, $\mathbb{Q}$ *and crises*

The measure $\mathbb{P}$, the objective or real-world measure, is also called the *historical measure* – it looks backwards. Prediction is irrelevant to this – and that is a serious matter when things are about to go seriously wrong, as in the build-up to and onset of a financial crisis. By contrast, the measure $\mathbb{Q}$, the risk-neutral measure, takes account of the *market*, because it deals with *prices*. Price is determined by *trading*, which involves a willing seller selling to a willing buyer – a highly non-trivial human interaction. Prices, and so $\mathbb{Q}$, are sensitive to *sentiment* – how a market (or, the market) is feeling collectively. This is a matter of psychology, and of confidence, as much as of objective fact. This is hardly surprising: money itself ultimately rests on confidence.

The difference between $\mathbb{P}$ and $\mathbb{Q}$ shows up dramatically at times of crisis. Perhaps the most spectacular crisis involving an individual firm was the collapse of Lehman Brothers on 14.9.2008. The credit *spreads* (differences) there between $\mathbb{P}$ and $\mathbb{Q}$ were dramatically large – and this is typical of what happens in a crisis. For, $\mathbb{P}$, the historical measure, looks backwards. But $\mathbb{Q}$, the risk-neutral measure, reflects *prices*, market *sentiment*, and *confidence* – all of which can change rapidly as a crisis develops!

3. "Big-picture stuff": MATL480 and MATL481, and beyond

The main result in MATL480 is the Black-Scholes formula, on the pricing of *options* on *stock* (part-ownership of firms, with prices quoted on the Stock Exchange). The main technical innovation (apart from Itô calculus, combining the power of calculus with that of probability) is that of $\mathbb{Q}$-measure (above). By contrast, MATL481 is about *interest rates*, the borrowing and lending of money, which takes place in the *bond markets*, or *money markets*, rather than *stock markets* as above. Essentially, the idea is to *break down the long time-periods involved into a number of shorter time-periods, each of which will have its own* $\mathbb{Q}$*-measure.*

Because the long-term future is impossible to predict with any certainty or even confidence, the decisions involved in the borrowing and lending of large sums of money over long periods of time — which are what determine interest rates — are similarly uncertain, and depend, not only on financial matters, but on economic and political matters. These interact with each other in complicated ways. Recall (MATL480 W0):

Anything important enough becomes political (Couve de Murville);

Politics is not an exact science (Bismarck);

Mathematics is an exact science. This is a mathematics course.

The "big picture", the media, and you.

We repeat something we said in MATL480 W0:

"To take a well-informed view of the 'big picture' — particularly the ongoing consequences of the Crash of $2007/8/\cdots$ — you need *background.* This cannot be acquired in a hurry. The best way to do this is to *keep abreast of current affairs*, by reading a good newspaper (or online equivalent), watching/listening to the news (especially political, economic, financial, $\cdots$) on television/radio, etc. Only you can do this! [Possibilities: daily, the FT (Financial Times); the Guardian, Times, Independent, Telegraph; weekly: Guardian Weekend, etc.; Sunday: Observer, Sunday Times etc.] This course [MATL480] will teach you Black-Scholes theory and (some) insurance mathematics. You will need to do the above for yourself to be able to put this in context and make the best use of it in later life". Similarly for MATL481!

LIBOR and SONIA

Recall (MATL480, I.1 W1a; see also I.4 W1b below) the London Inter-Bank Offer Rate (LIBOR). Bank Rate is the rate at which the Bank of England (BoE) lends to banks. It is changed fairly rarely, by BoE in consultation with the Treasury as representing the Government (HMG). As is well known, it has been at historically low levels (< 1 %) since the Crash ten years ago; it will be treated as riskless here. LIBOR is one of the most widely used measures of the rate at which banks lend to each other. It changes daily, is higher than Bank Rate, and is available over a number of time-periods (terms). Although there is no such thing as an absolutely riskless rate of interest, nevertheless LIBOR serves as a benchmark for one (the r in the Spot-Rate Models of Ch. III; cf. the LIBOR Market Models of Ch. V.3).

However, as we saw in MATL480, there was a major and very damaging scandal involving rigging of LIBOR (illicit market manipulation – 'collusion pretending to be competition'), inevitably known as the "Lie-bor" scandal. Hardly surprisingly, this led to plans to replace LIBOR, by something less vulnerable to such market manipulation – SONIA (Sterling Overnight Index Average).

SONIA 'is the effective reference for overnight indexed swaps for unsecured transactions in the Sterling market'. It was introduced in March 1997. The Bank of England (BoE) became its administrator in April 2016; it was reformed in April 2018. For details on its uses, see the BoE website (Google 'SONIA').

As we shall see (Part II, Weeks 4-7), the main part of the course is on *market models*. These split into two parts (the two theories are incompatible, but 'co-exist peacefully': cf. General Relativity and Quantum Theory in Physics). These are the LIBOR market models (LMM), and swap market models (SMM). As and when SONIA replaces LIBOR in common usage, these will need re-naming, e.g. SOMM and SwaMM. But we will stick with the names LMM and SMM for now.

Where to invest: stock or bonds?

Governments typically spend (health, education, defence, transport, police etc.) more than they take in in taxes. The difference is the Public Sector Borrowing Requirement (PSBR). This has to be made up by Government borrowing, added to the National Debt. This leads to the *bond market* for Government bonds ('gilts'). There are also bond markets for companies, which seek to raise capital for investment, by borrowing from banks (or by issuing new shares for investors). And there is the inter-bank borrowing market (LIBOR/SONIA above, etc.) For us, at the transition from MATL480 to MATL481, the most important thing is the *interplay between the stock and bond markets*, as they compete for investment capital. In former times, there were restrictions on the export of capital across state boundaries (reasonably: the UK currency is sterling; this has the Queen's head on it as UK head of state, etc.) In recent decades, globalisation (below) has led to enormous volumes of capital as 'hot money', constantly looking for maximum return in an ever-changing world. The result has been tsunamis of hot money flooding between national economies and between currencies, depending on the ebb and flow of economic/political events, nationally and globally. This can be profoundly destabilising, and is a major source of the uncertainty in the world today (political, financial, economic etc.) We consider some aspects of all this below.

Brexit.

One of the main sources of uncertainty in the UK over the next few years is of course Brexit. In particular, the future of London as a financial centre is uncertain. On the one hand, it is widely expected that London will remain 'Europe's banker', as it is easier and cheaper for companies to raise money in London than in the continental financial centres (mainly Frankfurt and Paris, also Zürich). On the other hand, most large international firms, including financial ones, have contingency plans, involving taking part of their operations overseas to remain within the EU.

Usually, the end-of-year financial assessments in the relevant newspapers (FT, etc. – see above for others) gives a good guide to the state of the economy in general and to that of the stock and bond markets in particular. This is less true now (2018-19) than usual, because of uncertainty relating to Brexit. The newspapers, radio, TV etc. are full of this. I hope you are following it! I know everyone is sick of it, including me. But, whether we like it or not (and I don't), this and related issues are going to dominate the financial scene for the foreseeable future. So anyone interested in a financial career needs to be able to discuss these things intelligently, e.g. at interview. The main questions as of now include:

1. Will there actually be a Brexit? — i.e., will the UK leave the EU?

2. If so, on what terms? The UK would then have a land border with the EU in Ireland. It is a vital part of the Good Friday Agreement, that ended the 30-year Troubles in Northern Ireland (NI), that there be no 'hard border' between (the Republic of) Ireland and NI. The UK would then also have a sea border with the EU in the English Channel. The present free flow of goods in large lorries through the Channel Tunnel ('Chunnel') would then be impeded, and Customs checks at the Channel ports — Dover and Calais — could result in massive delays and tail-backs.

3. Deal or No Deal? If Brexit takes place, will there be a UK-EU deal as part of the 'divorce settlement'? If so, will it resemble the EU-Norway or EU-Canada arrangements, or be a 'one-off'? (This would take years to arrange, and time is short.)

4. Delay or No Delay? The UK triggered Article 50, announcing its intention to leave the EU and 'setting the clock ticking'. The last $2\frac{1}{2}$ years of detailed negotiations are widely regarded as having been useless (or worse, as they have 'run the clock down').