MATL481 EXAMINATION 2017-18

Two and a half hours; six questions; do four; 25 marks per question.

Q1. Comment briefly on:
(i) Overnight index swaps;
(ii) Inflation;
(iii) Deflation;
(iv) Monetary policy.

Q2. (i) Define the LIBOR rate $L(t, T)$ at time t with maturity T, and show its inverse relationship with the zero-coupon bond price $P(t, T)$.
(ii) What is quantitative easing (QE)? Why is it used? Comment on its effects.

Q3. The *Theta*, Θ, of an option is defined as the time-derivative of its value.
(i) Given the Black-Scholes formula for the price c_t of European calls,

$$c_t = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2),$$

with S_t the stock price at time $t \in [0, T]$, K the strike price, r the riskless interest rate, σ the volatility and

$$d_{1,2} := [\log(S/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{T-t} : \qquad d_2 = d_1 - \sigma\sqrt{T-t} :$$

(a) find Θ and show that $\Theta < 0$;
(b) interpret this.
(ii) Given the corresponding Black-Scholes formula for the price p_t of European puts,

$$p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t\Phi(-d_1),$$

(a) find Θ, and show that this time Θ can change sign.
(b) Describe the conditions under which Θ will be positive, and interpret this.
You may quote that $Ke^{-r(T-t)}\phi(d_2) = S\phi(d_1)$.

Q4. (i) Give the stochastic differential equations (SDEs) for the Vasicek model for the spot rate $r = (r_t)$, under the real (objective) measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{P}^*$ (also known as $\mathbb{Q}^0$ and $\mathbb{Q}$ respectively). Show

how to pass between these SDEs.
(ii) Describe situations where each measure (and SDE) is useful.
(iii) Describe suitable data sets for use in each case, and (without proof) suitable statistical methods for handling them.

Q5. (i) Define the *forward rate* $f(t, T)$.
(ii) Describe the Heath-Jarrow-Morton (HJM) model for $f(t, T)$.
(iii) Describe the HJM drift condition.
(iv) What methods of proof are used here?

Q6. (i) Write down the price $P(0, T)$ of a non-defaultable zero-coupon bond in terms of the short rate of interest $r = (r_t)$.
(ii) In the defaultable case, with default intensity $\lambda = (\lambda_t)$ independent of r, show that the price $\overline{P}(0, T)$ is obtained from that of $P(0, T)$ by replacing r by $r + \lambda$.

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