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MATL481 INTEREST RATE THEORY: MOCK EXAMINATION 2017

Q1. Comment briefly on:
(i) the business cycle;
(ii) the Crash;
(iii) quantitative easing (QE);
(iv) persistent depression.

Q2. Given the stochastic differential equation

$$dF(t;T_1,T_2) = \sigma_2(t)F(t;T_1,T_2)dW_2(t), \qquad (LMM)$$

show that

$$F_2(T_1) = F_2(0)e^{m+VZ}, \qquad Z \sim N(0,1),$$

where

$$m = -\frac{1}{2}\int_0^{T_1} \sigma_2(t)^2 dt, \qquad V^2 = \int_0^{T_1} \sigma_2(t)^2 dt.$$

Q3. (i) Describe the two forms of the Schoenmakers-Coffey parametrisation for correlations.
(ii) Outline some of their desirable properties.
(iii) Give a two-parameter example.

Q4. Given that

$$v(t,x) := E[h(F_T)|F_t = x],$$

satisfies the Fokker-Planck equation

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma(t,x)^2 x^2 \frac{\partial^2 v}{\partial x^2} = 0, \qquad v(T,x) = h(x), \qquad (FoPl)$$

and writing $\phi(t,x)$ for the density of F_t, show that, for h a call C with strike K,

$$C(T,K) = \frac{1}{2}\sigma(T,K)^2 K^2 \phi(T,K).$$

That is, the *local volatility* $\sigma(T, K)$ is completely specified by the *volatility surface* $\sigma(K, T)$ by Dupire's formula,

$$\sigma(T, K) = \frac{1}{K}\sqrt{\frac{2\partial C(T, K)/\partial T}{\partial^2 C(T, K)/\partial K^2}}. \qquad (Dup)$$

Q5. Write down the price $P(0, T)$ of a non-defaultable zero-coupon bond in terms of the short rate of interest $r = (r_t)$.

In the defaultable case, with default intensity $\lambda = (\lambda_t)$ independent of r, show that the price $\overline{P}(0, T)$ is obtained from that of $P(0, T)$ by replacing r by $r + \lambda$.

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