ullintprob4

PROBLEMS 4, Week 5, 28.2.2018

Q1. *Rho*

Recall the 'Greek' rho, ρ, the sensitivity of an option price w.r.t. the risk-free interest rate r: for calls and puts,

$$\rho := \partial C/\partial r, \qquad \partial P/\partial r.$$

Recall also, from the Black-Scholes formula, with notation as there,

$$d_{1,2} := [\log(S/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{T-t},$$

and the standard normal density

$$\phi(x) := \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}.$$

(i) By exponentiating the definitions of d_1, d_2, show that (with $\tau := T - t$)

$$Ke^{-r\tau}\phi(d_2) = S\phi(d_1). \qquad (*)$$

(ii) Hence show that, for calls,

$$\rho > 0.$$

(iii) Give the financial interpretation of this.
(iv) Show that for puts,

$$\rho < 0.$$

(v) Give the financial interpretation of this.
(vi) Does this extend to American options? If so, prove it.

NHB