

MATL481 INTEREST RATE THEORY: RESIT EXAM 2017

Five questions; twenty marks per question

- Q1. (i) Define a zero-coupon bond (ZCB), and express its price $P(t, T)$ in terms of the spot rate r_t .
(ii) Define the (spot)-LIBOR rate $L(t, T)$.
(iii) Show that when LIBOR goes up, bond prices go down, and vice versa.
(iv) Explain what is meant by quantitative easing (QE), and the role of the result of (iii) in QE.

- Q2. (i) Define an affine term-structure model (ATM) for bond prices $P(t, T)$ in terms of the spot rate r_t .
(ii) Express this in terms of the instantaneous forward rates $f(t, T)$.
(iii) Show that the volatilities σ_f for f and σ for r are linked by

$$\sigma_f(t, T) = \frac{\partial B(t, T)}{\partial T} \sigma(t, r_t),$$

explaining what $B(t, T)$ here is.

- (iv) Show that if the coefficients in the dynamics of r_t are themselves affine functions of r , then one obtains an ATM for the forward rates f .
(v) Obtain the Vasicek model as a special case.

- Q3. (i) Define the Heath-Jarrow-Morton (HJM) model for forward rates $f(t, T)$.
(ii) State without proof the HJM drift condition.
(iii) Show that if the volatility $\sigma(t, T)$ of f is deterministic, then the forward rates f are Gaussian.
(iv) Show that this holds also for the short rate r .

- Q4. (i) Briefly describe market models.
(ii) Why are market models so important, and how do they compare with short-rate and forward-rate models?
(iii) Define Schoenmakers-Coffey (SC) matrices. What property do these have that makes them correlation matrices?
(iv) Explain why SC matrices show increasing correlation between adjacent rates as the tenor increases, and why this is a realistic feature of forward rates.

- Q5. Comment briefly on:
- (i) collateralized debt obligations (CDOs);
 - (ii) toxic debt;
 - (iii) securitization;
 - (iv) negative interest rates.

N. H. Bingham