ullinttest2

TEST 2. Week 6, 7.3.2018

Theta

The *Theta*, Θ, of an option is defined as the time-derivative of its value.
(i) Given the Black-Scholes formula for the price c_t of European calls,

$$c_t = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2),$$

with S_t the stock price at time $t \in [0,T]$, K the strike price, r the riskless interest rate, σ the volatility and

$$d_{1,2} := [\log(S/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{T-t}: \qquad d_2 = d_1 - \sigma\sqrt{T-t}:$$

(a) find Θ and show that $\Theta < 0$;
(b) interpret this.
(ii) Given the corresponding Black-Scholes formula for the price p_t of European puts,

$$p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t\Phi(-d_1),$$

(a) find Θ, and show that this time Θ can change sign.
(b) Describe the conditions under which Θ will be positive, and interpret this.
You may quote that $Ke^{-r(T-t)}\phi(d_2) = S\phi(d_1)$.

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