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MATL480: STOCHASTIC MODELLING IN INSURANCE AND FINANCE

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Wednesday 27 September 2017

This course will be taught on seven consecutive Wednesdays, 27 September to 8 November, all day 10am to 6pm, with 3 hours of lectures am, 10-1 (W1a - W7a), an hour for lunch 1-2pm, three hours of lectures pm, 2-5 (W1b - W7b), and an examples class 5-6pm. These will be open for student questions, and will also cover Problems and Solutions 1a, 1b - 7a, 7b.

In the absence of scheduled Office Hours (my office is at Imperial College London, not ULL) I will also be available during the lunch breaks, to any student who wants to see me (and doesn't mind my talking while eating! This works well: I have working lunches with collaborators and research students.)

Course texts: For Ch. I-VI (Mathematical Finance): Ch. 1-6 of
[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., CUP, 2004.
For Ch. VII (Insurance Mathematics): Part I (Ch. 1-4) of
[Mik] Thomas MIKOSCH, *Non-life insurance mathematics: An introduction with stochastic processes*, Universitext, Springer, 2004.

Course Website: ULL (MATL480) link on my Imperial College homepage (Imperial College > Mathematics Department > Staff > Staff List > Bingham > Homepage) – or, just Google Nick (or Nicholas, or N H) Bingham. *Favouritise this* to get it in one click.

CONTENTS

§3. Brief history of mathematical finance.
§4. Markets and options.
§5. Portfolios and hedging.
§6. Arbitrage.
§7. Put-call parity.
§8. An example.
§9. Complements
Prelude to Ch. II

II. PROBABILITY BACKGROUND [$\frac{3}{4}$ week: W2a, 2b].
§1. Measure.
§2. Integral.
§3. Probability.
§4. Equivalent measures and Radon-Nikodym derivatives.
§5. Conditional expectations.
§6. Properties of conditional expectations.

III. STOCHASTIC PROCESSES IN DISCRETE TIME [$\frac{1}{2}$ week: W2b,3a].
§1. Filtrations and information flow.
§2. Discrete-parameter stochastic processes.
§3. Discrete-parameter martingales.
§4. Martingale convergence.
§5. Martingale transforms.
§6. Stopping times and optional stopping.
§7. The Snell envelope and optimal stopping.
§8. Doob decomposition.
§9. Examples.

IV. MATHEMATICAL FINANCE IN DISCRETE TIME [$1\frac{1}{4}$ weeks: W3a-4a].
§1. The model.
§2. Viability: existence of equivalent martingale measures (EMMs).
§3. Complete markets: uniqueness of equivalent martingale measures.
§4. The Fundamental Theorem of Asset Pricing: Risk-Neutral Valuation.
§5. European options. The discrete Black-Scholes formula.
§6. Continuous-time limit of the binomial model.
§7. More on European options.

§8. American options.

V. STOCHASTIC PROCESSES IN CONTINUOUS TIME [$\frac{3}{4}$ week: W4a-5a].
§1. Filtrations; finite-dimensional distributions.
§2. Classes of processes.
§3. Brownian motion.
§4. Quadratic variation of Brownian motion.
§5. Stochastic integrals; Itô calculus.
§6. Stochastic differential equations; Itô's Lemma.

VI. MATH. FINANCE IN CONTINUOUS TIME [$1\frac{1}{4}$ weeks: W5a-6a].
§1. Geometric Brownian motion and asset prices.
§2. The Black-Scholes model.
§3. The (continuous) Black-Scholes formula via Girsanov's theorem.
§4. The Black-Scholes formula via the Black-Scholes PDE and the Feynman-Kac formula.
§5. Infinite time-horizon; American puts.
§6. Real Options (Investment Options).
§7. Further results. Postscript to Ch. I-VI.

VII. INSURANCE MATHEMATICS [$1\frac{1}{2}$ weeks: W6b - 7b]
§1. Insurance background.
§2. The Poisson process; compound Poisson processes.
§3. Renewal theory.
§4. The ruin problem.
§5. Lundberg's inequality.
§6. The ruin problem and the renewal equation.
§7. Cramér's estimate of ruin.
§8. Complements.
§9. More on insurance; Postscript to Insurance.

Division of Time
Week 1a: I, §1-5 [28 Sep]
Week 1b: I, §5-9; Prelude to Ch. II [28 Sep]
Week 2a: II, §1-5 [5 Oct]
Week 2b: II, §5-6; III, 1-6 [5 Oct]

Week 3a: III, §6-9; IV, 1-2 [12 Oct]
Week 3b: IV, §3-6 [12 Oct]
Week 4a: IV, §7-8; V, 1-2 [19 Oct]
Week 4b: V, §2-5 [19 Oct]
Week 5a: V, §5-6; VI, 1-3 [26 Oct]
Week 5b: VI, §3-5 [26 Oct]
Week 6a: VI, §6-7; Postscript [2 Nov]
Week 6b: VII, §1,2 [2 Nov]
Week 7a: VII, §3,4 [9 Nov]
Week 7b: VII, §4,5 [9 Nov]

Format and Examination.

Lecture times as above. The material will be in 14 parts (1a,b – 7a,b), of around 12 pages each. I aim to cover about 4 pages per hour, and have time for questions. There will be Problems am and pm each week, with Solutions the week after. The syllabus is unchanged from last year; Ch. I-VI is that of Math482, which I taught at the University of Liverpool for four years 2013-16. See the MATL480 link on my homepage for Exam + Solutions 2016, and Mock Exam + Solutions (and for info, the Math482 link for Exam + Solutions 2013 – 2016 and Mock Exam + Solutions).

Background and general interest

[K1] John KAY, *The Truth about Markets: Their Genius, their Limits, their Follies*. Penguin/Allen Lane, 2003.

[K2] John KAY, *Other People's Money: Finance: Masters of the Universe or Servants of the People?* Profile Books, 2015, £16.99.

[AH] Anat R. ADMATI & Martin HELLWIG, *The Bankers' New Clothes: What's Wrong with Banking and What to Do about it*, Princeton UP, 2013.

[G] Alan GREENSPAN, *The age of turbulence*. Penguin, 2007.

I think the Kay books are essential reading for anyone thinking of working in the financial services industry – maybe [AH] too. I thoroughly recommend [G] – but get the latest edition of it that you can. The author was Chairman of the US Federal Reserve (Fed) 1987-2006. His views up to 2007 were largely Panglossian optimism ("markets know best, and are self-correcting", etc.). The ongoing problems since have forced a re-think; see the epilogues to later editions, his evidence to the House Committee, etc.

Books for reference.

[BF] N. H. BINGHAM and J. M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), Springer, 2010 (regression is about *conditioning* – informally, using what we know).
[CR] John C. COX and Mark RUBINSTEIN: *Options markets*. Prentice Hall, 1985.
[H1] HULL, J. (1995): *Introduction to futures and options markets* (2nd ed), Prentice-Hall, ('baby Hull'), or
[H2] HULL, J. (1993): *Options, futures and other derivative securities* (2nd ed.), Prentice-Hall ('Hull').
[DP] Avinash K. DIXIT and Robert S. PINDYCK: *Investment under uncertainty*. Princeton University press, 1994.
[PS] G. PESKIR and A. N. SHIRYAEV: *Optimal stopping and free-boundary problems*. Birkhäuser, 2006.

Mathematics, for reference
[D] J. L. DOOB: *Stochastic processes*, Wiley, 1953.
[N] J. NEVEU: *Discrete-parameter martingales*, North-Holland, 1975.
[KS] KARATZAS, I. & SHREVE, S. (1988): *Brownian motion and stochastic calculus*. Graduate Texts in Math. **113**, Springer.
[RY] REVUZ, D. & YOR, M. (1999): *Continuous martingales and Brownian motion*. Grundlehren der math. Wiss. **293**, Springer, 3rd ed. (1st ed. 1991, 2nd ed. 1994,).
[RW1] ROGERS, L. C. G. & WILLIAMS, D. (1994): *Diffusions, Markov processes and martingales, Vol. 1: Foundations*, 2nd ed., Wiley (1st ed. 1970).
[RW2] ROGERS, L. C. G. & WILLIAMS, D. (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*. Wiley.

Mathematics for reference: Insurance Mathematics
[A] S. ASMUSSEN, *Applied probability and queues*, 2nd ed., Springer, 2003 [1st ed. Wiley 1987].
[AA] S. ASMUSSEN and H. ALBRECHER, *Ruin probabilities*, 2nd ed., World Scientific, 2010 [1st ed., S. Asmussen, 2000].
[Kyp] Andreas E. Kyprianou, *Fluctuations of Lévy processes with applications: Introductory lectures*, 2nd ed., Springer, 2014 [1st ed. 2006].
[RSST] T. ROLSKI, H. SCHMIDLI, V. SCHMIDT and J. L. TEUGELS, *Stochastic processes for insurance and finance*, Wiley, 1999.

The Big Picture

Please bear three things together in mind throughout this course:
1. Anything important enough becomes political (M. Maurice Couve de Murville). This stuff is certainly important.
2. Politics in not an exact science (Bismarck). But,
3. Mathematics is an exact science.
We will be doing lots of mathematics – in particular, we derive the Black-Scholes formula. We will extend calculus, the most powerful single weapon we have, to become probabilistic (Itô calculus) and apply it to these problems. But, there are limits to which finance, economics, or anything involving human psychology, is mathematicisable. As always in Applied Mathematics, we have to be on guard: if we don't simplify enough, we can't do anything; if we over-simplify, we can do things, but can't trust our conclusions.

Just as important as the technical mathematics, you need to think about the systemic faults at the geofinancial/economic/political level thrown up by the crisis of 2007 on (Credit Crunch, etc.). Any prospective employer in the financial services industry should ask you questions about this, and your views on it, in interview. See e.g. the books cited above [K1], [K2], [AH], [G], etc. Some of my views are in
N. H. BINGHAM: The Crash of 2008: A mathematician's view. *Significance* **5** no. 4 (2008), 173-175.
For some of my further views, see my homepage, Talks > Crash.

Two more books (general interest).
Michael LEWIS, *Liar's Poker: Rising through the Wreckage on Wall Street*, 1989, W. W. Norton & Co.
Michael LEWIS, *The Big Short: Inside the Doomsday Machine*, 2010, W. W. Norton & Co. (filmed, 2016).

Life after this course.

To take a well-informed view of the 'big picture' – particularly the ongoing consequences of the Crash of 2007/8/... – you need *background*. This cannot be acquired in a hurry. The best way to do this is to keep abreast of current affairs, by reading a good newspaper (or online equivalent), watching/listening to the news (especially political, economic, financial, ...) on television/radio, etc. Only you can do this! [Possibilities: daily, the FT (Financial Times); the Guardian, Times, Independent,Telegraph; weekly: Guardian Weekend, etc.; Sunday: Observer, Sunday Times etc.] This course

will teach you Black-Scholes theory and (some) insurance mathematics. You will need to do the above for yourself to be able to put this in context and make the best use of it in later life. NHB