MATL480 EXAMINATION 2016

Q1. In the context of insurance, comment briefly on:
(i) limited liability;
(ii) reinsurance;
(iii) regulation;
(iv) lender of last resort.

Q2. (i) Define volatility.
(ii) Comment briefly on: historic volatility; implied volatility; the volatility surface.
(iii) How do option prices depend on volatility, and why?
(iv) Discuss the effect of trading volume on volatility, and its implications for market stability.

Q3. The current price of Brent crude oil is \$ 150 per barrel. Next year, the price will be up to 153 or down to 144, each with positive probability. Neglect interest.
(i) Price a call option C for a barrel of Brent crude oil next year, with strike price K the current price 150.
(ii) Hedge this option.
(iii) What are some of the relevant factors that will determine whether oil prices next year are up or down?

Q4. (i) State without proof the Paley-Wiener-Zygmund theorem, constructing Brownian motion $W = (W_t)$ via an expansion in Schauder functions $(\Delta_n(t))$.
(ii) Show that Brownian motion is scale-invariant: for $c > 0$, $W_c = (W_c(t))$ is again Brownian motion, where $W_c(t) := W(c^2 t)/c$.
(iii) What does this tell us about the limitations of Brownian motion as a model for driving noise in the Black-Scholes model?

Q5. (i) For the Black-Scholes model, give the stochastic differential equation for geometric Brownian motion, and solve it.
(ii) Show that both returns over $(t, t + dt)$ and log-prices are normally distributed.
(iii) What happens in two dimensions? With correlation ρ, consider both $\rho > 0$ (two stocks in the same sector, say) and $\rho < 0$ (two stocks in different

sectors).

Q6. (i) Define the Poisson process $N = (N_t)$ with rate λ, and the compound Poisson process $S = (S_t)$ with rate λ and jump-distribution F, $CP(\lambda, F)$.
(ii) For $CP(\lambda, F)$, find the characteristic function of S_t.
(iii) Find the mean and variance of S_t, when F has mean μ and variance σ^2.
(iv) Show that with λt large, S_t is approximately normally distributed.

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