

PROBLEMS 3b. 11.10.2017

Q1. *The doubling strategy.*

Analyze the ‘doubling strategy’: when betting on tossing a fair coin, respond to losing by doubling the stakes.

- (i) Show that an eventual win is certain.
- (ii) Show that this certain gain is 1.
- (iii) Show that the mean time to this win is 1.
- (iv) Explain why this is an impossible (or at least suicidal) strategy to follow in practice.

Q2. *Simple random walk (SRW).*

We bet on independent tosses of a fair coin. Our strategy is to bet until we are first ahead, and then quit. Let T be the time we quit (a stopping time). Show that:

- (i) T has PGF

$$E[s^T] = \frac{1 - \sqrt{1 - s^2}}{s}$$

(hint: look at what happens if we play until we are 2 ahead for the first time);

- (i) $T < \infty$ a.s.: we are certain of achieving a net gain of +1 eventually;
- (ii) $ET = +\infty$: the mean waiting time till this happens is infinite;
- (iii) this strategy too is impossible in practice.

Recommended Reading: Grimmett & Stirzaker [GS], §5.2 (Bingham & Kiesel Ex. 3.4).

NHB