MATL480 EXAMINATION 2017-18

Q1. Comment briefly on:
(i) sub-prime mortgages;
(ii) asset-price bubbles;
(iii) quantitative easing.

Q2.(i) What is meant by hedging? Who hedges, and why?
(ii) Discuss briefly the most important types of hedging.
(iii) Discuss briefly hedging in discrete time and in continuous time, and their contrasts.
(iv) Should one hedge not at all, partially, or completely, and why?

Q3. In a two-period binary model, at each node the stock goes up by a factor of 5/4 or down by a factor of 4/5, each with positive probability. The payoff is $(S-8)_+$, with S the final stock-price; the initial stock-price is 8. Neglect interest.
(i) Find the martingale probability p^* that the stock goes up.
(ii) Working down the tree, find the value of the option at each node.
(iii) Working up the tree, find the hedging portfolio at the time-0 and time-1 nodes.

Q4. The partial derivative of an option price w.r.t. the volatility σ is called its *vega*, v.
(i) For European calls and puts in the Black-Scholes model, show that vega is positive.
(ii) By using (i) and the Snell envelope, or otherwise, show that vega is also positive for American options.

Q5. (i) Give the stochastic differential equation for $S = (S_t)$ geometric Brownian motion $GBM(\mu, \sigma)$ with parameters μ and σ. State its solution, without proof.
(ii) State the risk-neutral valuation formula (in continuous time), applied to a European call option with stock price S_t at time $t \in [0, T]$, strike price K, riskless interest rate r, volatility σ and expiry T.
(iii) Hence or otherwise derive the Black-Scholes formula for the price of the

call at time $t = 0$:

$$c_0 = S_0 \Phi(d_+) - K e^{-rT} \Phi(d_-), \quad d_{\pm} := [\log(S/K) + (r \pm \frac{1}{2}\sigma^2)T]/\sigma\sqrt{T}. \quad (BS)$$

Q6. (i) For the exponential law $E(\lambda)$ with parameter $\lambda > 0$ (density $f(x) = \lambda e^{-\lambda x}$ for $x > 0$, 0 otherwise), show that the renewal function is

$$U(x) = 1 + \lambda x.$$

Interpret the two terms in this result.
(ii) Show that this agrees with the renewal theorem, Blackwell's renewal theorem and the key renewal theorem in the $E(\lambda)$ case.

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