RISK FOR ACTUARIES AND RISK FOR EVERYONE

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This talk will be deliberately discursive – partly because the audience is a general one. It will be deliberately wide-ranging – the point being that risk is everywhere, and that a proper understanding, weighing and balancing of the risks we are exposed to, as individuals and as a society is of prime and growing importance.

If it be now, 'tis not to come; if it be not to come, it will be now; if it be not now, yet it will come; the readiness is all: ... [*Hamlet*, V.II].

The essence of the actuarial profession is the quantitative aspects of insurance, and the essence of insurance is the sharing of risk.

The founding father here is arguably John Graunt (1620-1674), with his *Bills of Mortality* in 1662 (a little after the Pascal-Fermat correspondence of 1654, which marks the 'official beginning' of probability theory). Equitable Life was founded in London in 1762 (one of the world's oldest insurance companies, it nearly collapsed and closed to new business in 2000; an Equitable Life Bill was announced in 2010). Scottish Widows began in 1812 (demutualised and became part of Lloyds TSB in 2000).

In life insurance, the time-frame is very long; the claim streams depend on future deaths, and so are uncertain; the income stream depends on future subscribers, and the future of investments, both uncertain. So the scope for getting it wrong is vast (witness Equitable Life!) Getting it right depends on a great deal of mathematics, probability, statistics (and nowadays computing). This is an area with a future – and a socially useful one.

Why insure?

I know a property developer who never insures his properties. As he reasons, the worst that could happen is that one of them might burn down. If it did, he would himself rebuild it (or sell the plot for redevelopment); meanwhile, he saves the premiums.

Most people do insure. Unlike him, they have only one property (loss of

which would be disastrous), and lack the professional knowledge to build a house themselves. For such people, insurance premiums are a heavy burden – but one they voluntarily assume, indeed in the hope and expectation that they will *not* get something back for it (because they don't want the inconvenience of the accident that leads to the claim).

From the insurance company's point of view (and here is where the actuarial angle comes in), what counts is choosing one's premium levels sensibly. If these are too high, one loses business to one's cheaper competitors, and goes out of business. If they are too low, one does not have the financial reserves to meet a run of bad luck in which unusually many or high claims arrive, and one goes bankrupt. It is one of the prime jobs of an actuary to advise the company here, on the basis of the information available.

Insurance and utility

To a rich man, a pound is worth much less than to a poor man; to a very rich man, a six-figure sum is modest, rather than the answer to all one's prayers. This is an instance of the *law of diminishing returns*. To formalise this, one introduces a *utility function*, U. This is non-negative, but (because of the law of diminishing returns, or *satiation*, often bounded. It is concave ($U'' \leq 0$ – curve bends downwards, below its tangent at the origin). Near the origin, the curve is approximated by its tangent, so straight – utility = cash. Far to the right, curvature predominates. Because the company is big and the individual customer is small, the premium for his house is on the straight part of the company's utility curve (it's just money to them), but on the curved part of the customer's utility (as having his house burn down uninsured is too ghastly to contemplate). It is this disparity that makes it possible for a premium level to be fixed that both parties are satisfied with, and this makes insurance possible.

Juvenal's question

Quis custodiet ipsos custodes? (Juvenal, *Satires*, c. 100 AD). Who guards the guards? Who polices the police? Who insures the insurers? Who reinsures the reinsurers?

Juvenal's question is as sharp today as it was in antiquity – and as it always will be – partly because it has no answer. What it does is to force one to think about fundamental questions regarding society. There are no definitive answers here (people individually are complicated enough, people collectively even more so), but what answers there are are ultimately political (Everything important enough becomes political (Couve de Murville); Politics is not an exact science (Bismarck)).

Another aspect of this was well put by Keynes long ago (J. M. KEYNES (1883-1946)): if you owe the bank 5 thousand pounds, that's your problem; if you owe it 500 thousand pounds, it's the bank's problem.

This raises a multiplicity of questions. One is one the issue of banks being "too big to fail" (e.g., in the UK, RBS among others, ongoing, and – in the US and the UK – the separation or not between high-street/retail banking and investment/merchant banking). Another is the question of those facing risk they are unwilling or unable to bear: where to lay it off to. Insurance companies insure themselves against really large risks. This is *reinsurance*. But, for an ordinary client, what happens if your insurance company goes bust? – and, for an insurance company, what happens if your re-insurance company goes bust? – etc.

The phrase that comes to mind here is that of the *lender of last resort*. In banking, in the UK, the lender of last resort is the Bank of England; in the US, it is the Fed. Society is too complex, and too inter-connected, for governments to stand by and take a detached view of the failure of large banks, insurance companies etc., who will take down with them the savings of large numbers of people (who are electors!), who have invested with them in good faith, trusting that what should be regulated is regulated, appropriately – that is, reassured by Government guarantees, explicit or implicit.

In passing: the relevant mathematics here covers the "tails" of the relevant probability distributions – quantifying the probability of large, or extremely large, losses. The tails are decisive in limit theorems in probability theory, the kind of thing I do for a living. They are also decisive in Extreme-Value Theory (EVT), the speciality of my old friend Paul Embrechts of ETH-Zürich, one of my predecessors as a speaker on this occasion.

Old hands will recall various events. LTCM (Long-Term Capital Management) was bailed out by Mr Alan Greenspan in 1998 when he was Chairman of the Fed, essentially because he saw it as "too big to fail". This led to the "Greenspan put": the belief among big market agents that one could gamble with a degree of immunity – if it worked, the gains were yours; if it didn't, the losses would be covered by Greenspan in the interests of market stability. This was one of the drivers of the asset price bubble that burst with the US sub-prime mortgage scandal of 2007 and the collapse of Lehman Brothers in 2008, with fallout in the UK with Northern Rock in 2008, and is ongoing in the eurozone and other crises of today.

HMQ's question

On 5 November 2008, Her Majesty the Queen officially opened the LSE's

New Academic Building, just round the corner. As this is the London School of Economics and Political Science, and as the financial crisis affects her subjects, Her Majesty memorably asked her host on this occasion: *Why weren't we warned?* (a question that I hope duly makes it into the Oxford Dictionary of Quotations). This prompted a letter to HMQ from the British Academy of 22.7.2009. I urge you all to download it and read it (I'll read it myself here if I have time).

Intensity, rate and the Poisson process

Insurance tends to split, into life and non-life. In life insurance, the triggering events are deaths – certain eventually (see the above quotation from *Hamlet*), but quite unlikely in any individual time-interval (if reasonably short). In non-life insurance, the triggering events are accidents, etc. Such events happen 'out of the blue', unpredictably. To model them, one uses the *Poisson distribution* $P(\lambda)$ with *parameter* $\lambda > 0$:

$$P(X = k) = e^{-\lambda}\lambda^k/k! \qquad (k = 0, 1, 2, \ldots)$$

(S. D. POISSON (1781-1840) in 1837), which has mean λ and variance λ. This was used as a statistical model by L. J. BORTKIEWICZ (1868-1931) in 1898 (*Das Gesetz der kleinen Zahlen* = The law of small numbers) – to analyse deaths by kicks of a horse in the Prussian cavalry! All this is static; the dynamic view came in in the early 1900s. What we now call the *Poisson process* was introduced in 1903 by Filip LUNDBERG (1876-1965), a Swedish actuary, in the theory of collective risk, and again in 1909 by A. K. ERLANG (1878-1929), a Danish telephone engineer. One speaks nowadays of a *Poisson point process* $Ppp(\lambda)$ with *rate* or *intensity* λ. This usage did not exist at the time of the work of Lundberg and Erlang. The term *stochastic process* (stochastic meaning random, process meaning unfolding with time) was introduced by A. Ja. KHINCHIN (1894-1959) in the 1920s. The two prototypes – Brownian motion (Einstein in 1905), aka the Wiener process (Wiener in 1923), and the Poisson process – together with its extensions, the *compound Poisson processes* (which model accumulated claims to date – the claims are random, but occur at Poisson points) led to the Lévy-Khintchine formula and the class of Lévy processes of the 1930s.

With intensity λ, the probability of an event in an interval of length t is λt – or in $[t, t+dt]$ is λdt. This is small if t (or dt) is small. This means that one has an excellent chance of not dying in any specified (short) time-interval.

This even extends to situations of deliberate danger such as combat. With the exception of bloodbaths such as the first day of the Battle of the Somme,

the chances of an individual combatant surviving an individual action have usually been very good. What does the damage is repeated exposure: the attrition of the whole period of the Somme or of Passchendaele in WWI, or of a tour of duty in Bomber Command in WWII, rather than an individual raid.

The Poisson parameter λ can be thought of either as a rate – at which events (accidents, combat deaths, etc.) occur, or as an intensity (of the battle, say). Most people are very sensitive to the idea of exposure to risk, particularly is this is avoidable. Instead of 'I'll probably survive', one thinks 'I could be killed'. Doctors tend to be more relaxed – one has to die of something, sometime.

Medical issues

Medicine saves lives! Look at the increase in longevity over the last few centuries since the development of modern medicine, the drop in infant mortality, etc. But, medical progress depends on increasing medical knowledge, which (as elsewhere in science) involves experimentation – on whom?

New procedures have to be tried out – before it is known how they compare with the existing ones. So, some patients have to be treated one way and some another. One treatment will turn out to be better – which is tough on those who get the other one, particularly if the disease is life-threatening (which let us suppose it is, to bring the point home). The point is that the medical experimentation on which medical progress depends involves lives – quite large numbers of them, as trials need to be large for the conclusions to be reliable, with lives now (those who get the inferior treatment) being traded against lives in the future (which will be saved by the superior treatment). What is clear is that needs both great care in the design of the experiment (a subject of its own right in Statistics, with a subfield of Clinical Trials in Medical Statistics), and of ethical scrutiny (by properly responsible committees and professional bodies, backed up by legislation).

Note that, if undergoing surgery, one would prefer to be operated on by an experienced surgeon. But, surgeons necessarily begin as inexperienced. If we only use experienced surgeons, there will be no surgeons left to replace them when they die. The upshot is that there is a degree of risk balancing (see below), with some cost to those treated now to be offset against the gains to those to be treated in the future.

A propos of this – the inevitability of starting from scratch and a degree of 'learning on the job': this happens in any profession (bus drivers, airline pilots etc. – though pilots can train extensively on flight simulators before

flying real passengers). It happens in my profession, academia. I started as a lecturer in 1969. What I remember (I pass over my inevitable initial fumblings) was being trusted to do my job before I knew my job – which concentrated my mind wonderfully on doing so. I've been a professor for 27 years now; no one trusts me to do anything – but, no one trusts anyone to do anything nowadays. I will have to leave you to draw your own conclusions on the issues raised here.

Risk balancing

We have had publicity recently about the pros and cons of screening women for breast cancer. This hinged on the distress caused to women with false positives – who were picked up at screening, and treated, but who turned out not to have had the disease or needed the treatment given (breast surgery in many cases – in addition to the psychological distress caused). Against this, undetected breast cancer kills women; we probably all know of such cases. It is hardly possible for lay people to have a properly informed view in areas such as this, where medical opinion is itself divided, or is changing with time as medical knowledge expands (see above!).

Rather analogous problems arise regarding prostate cancer in men. The belief is common and growing that all men who live long enough get prostate cancer (not to mention benign prostatic hyperplasia or BHP – enlargement of the prostate, a natural ageing process). The disease is really a spectrum of diseases; the more malignant will kill; the less malignant are such that most men diagnosed with prostate cancer die with it rather than of it. Treatment can lead to unpleasant complications. Again, the balance of risks here is less clear than is comfortable.

It is accepted that partners of uncircumcised men are more at risk of cervical cancer than those of circumcised men. Circumcision (a practice that is thought to have originated for hygienic reasons) thus has respectable medical arguments in its favour. Against is the reluctance to interfere with Nature avoidably – and the mortality rate (small, but non-trivial!) from complications. This is predictable on statistical grounds alone. There is a mortality rate from removal of the tonsils, the appendix etc. – from any surgical procedure, involving as it does incision and bleeding. This is not to mention the mortality rate from childbirth – the most natural thing in the world.

Risk compensation

Statistically, questions of risk are complicated by the phenomenon of risk compensation. Those old enough will remember a safety campaign of perhaps twenty years ago. The poster depicted a young man on a motor-bike, with a

young woman pillion passenger with her arms around him, both stark naked (something there for everyone, regardless of gender or orientation!). The caption read (words to the effect that): If all motor-cyclists were equipped like this, casualties would be fewer. The word 'counter-intuitive' passes across the mind. After some thought, the penny drops. If one's flesh is so nakedly exposed to the danger of any accident, every fibre of one's being strains to prevent any accident - by scrupulous caution and extra care, by defensive driving/riding, by restraint on speeds, etc. Similar conclusions are forced on one if one analyzes the road-accident statistics following the introduction of seat-belts. Casualties fell dramatically – until drivers realized they were safer, and some of them responded to feeling safer by driving closer to the limits of their road-space. Similarly with the adoption by motor manufacturers more generally of the 'crumple-boxes' that were originally a distinctive safety feature of pioneering manufacturers such as Volvo. Again, in lifestyle choices generally: smoking has passed from being almost de rigeur among the young of my and my parents' generation – where it was regarded as a rite of passage of adulthood (I ducked out gracefully because I am a runner) – to being a minority indulgence, associated with the less intelligent and less educated end of the spectrum and subject to increasing legal restrictions in public. By way of compensation, binge drinking among the young – most worryingly, among young females – has mushroomed to epidemic and worrying proportions.

Public understanding of risk

The general public is not particularly interested in, or well-informed about, science in general, mathematics, probability and statistics, and in particular, risk. But we live in an increasingly complicated world, where a proper appreciation of such things is necessary in order to guide decisions we need to take. In an attempt to address this, in recent years chairs in leading universities have been endowed for public education in these matters. The first such was the chair in Public Understanding of Science, held at Oxford by Richard Dawkins from 1995-2008 (when he was succeeded by Marcus du Sautoy). This was followed by the chair in Public Understanding of Risk, held at Cambridge from 2007 on by David Spiegelhalter.

Governmental understanding of risk

It is not just the public that is under-aware of risk. You have probably seen the recent publicity about ash die-back, an unpleasant viral disease which may well devastate the UK's many and lovely ash trees, just as Dutch elm disease devastated our elms a generation ago. It seems that the virus

was imported into the UK in ash saplings from the Netherlands. Now any gardener will tell you that ash sprouts freely all over the place, and one has to yank out and destroy ash saplings all the time. But, this has not been organised to make the UK self-sufficient in ash, while the Netherlands has invested in growing ash (and no doubt other) saplings more cheaply than elsewhere. When passing through ports and airports, one passes signs warning one that unauthorised import of biological material is a criminal offence. No doubt the importation of ash saplings was duly authorised. It would of course not have been had the relevant officials had any inkling that this might happen. It is clearly a very false economy to import ash saplings at the expense of writing off much (I trust not all!) of one's ash stock. Ignorance is expensive.

One sees similar evidence of the authorities walking unsuspectingly into a minefield over the spread of infectious diseases in animals – and indeed, people. The foot-and-mouth disease outbreak in 2001 traumatised the farming community – and reduced Whitehall and Westminster to panic. It so happens that my college, Imperial, is a national centre for epidemiology. The services of the Imperial epidemiologists were much in demand (I wasn't there then, but one hears about these things). Similarly for HIV/AIDS, SARS, swine flu, bird flu, etc.

You may recall the ghastly period when people were dying of BSE (bovine spongiform encephalitis – "mad cow disease", and its human equivalent). We stopped eating beef when our medical friends stopped eating beef. One way to produce beef more cheaply is to give cattle protein in their feed. One source was mechanically recovered protein, from slaughtered animals. Cattle are ruminants – they feed on grass. Feeding cheap protein to cattle was asking for trouble; trouble duly came. As far as I am aware, what was done was not criminal – but it should have been. Practices grew up in the trade, which were sanctioned by government; the motivation was to save money; the risks should have been obvious, but were not; lives were lost. Again, ignorance is expensive – in lives, as well as money.

Health and Safety

Risk and safety go hand in hand; one can hear anecdotal evidence, and indignant denunciation, of the excesses of "elf 'n safety" in any bar. I heard a discussion quite recently of the effect of the legislative requirements on the areas involving children – teachers, priests, scoutmasters, sports coaches etc. I heard a discussion last year of firms discontinuing offering summer jobs to students because their legal advice was that such temporary staff should be accompanied at all times, or to such an extent that taking them on was

clearly not worthwhile. I have every sympathy with this. But, as you will be thinking before I say it, because the fallout from the Jimmy Savile case is all around us, people with a sexual interest in children – paedophiles – are attracted to professions where they will encounter children – and are extraordinarily manipulative, and skilled at evasion. I would rather live with H & S as it is here than go back to former, less regulated times.

In praise of H & S: I cannot resist mentioning some past successes here. In the early 19th century, Britain had a large and flourishing merchant navy; trade was the lifeblood of the Empire and the economy. Ships were routinely over-laden; over-laden ships would regularly sink in storms, with the loss of ship, cargo and crew. The owners didn't care: ships and cargo were insured (back to insurance!), and as for crew – ownera and officialdom took the view that this was an occupational hazard. Samuel Plimsoll (1824-1898) is commemorated in the Plimsoll Line (1876) – the line (which looks rather like the London Underground logo) on a ship's hull that marks the legal limit beyond which the ship may not be loaded. This took decades of struggle to achieve – and has saved countless lives.

I will give another example of H & S. I come from a farming area in the North of England, and was used to seeing farm tractors as a boy (indeed, I remember the horses and carts before the tractors). In the hilly parts of the North, tractors regularly rolled over when turned on a steep gradient – crushing the driver to death. This sickening waste of life was prevented when legislation made it mandatory for such tractors to be fitted with protective bars. Similarly for the introduction of the protective metal cladding to the sides of the trenches one sees being dug in roads – which used to collapse, and kill workmen.

Insurance and mis-selling

Back to insurance. The financial world has yet to recover from the crisis of 2007-8 (sub-prime mortgages and the collapse of Lehman Brothers in US, Northern Rock, RBS etc. in UK). Part of the build-up to this was the drive towards *securitisation* – basically, the drive to identify any risk, and turn it into a product, that could be bought, sold or traded. One can tell when things are getting out of hand: when the secondary market (in such derivatives) has outgrown the primary market (in the underlying real-world economy), sometimes by an order of magnitude.

The banking industry has played its part in giving insurance a bad name in all this. We are still experiencing the fall-out from the various mis-selling scandals of recent years (PPI – payment protection insurance: just Google

"PPI""scandal", if you can stomach it). The agressive mis-selling of artificial and unnecessary insurance products – motivated by money, and exploiting the lack of knowledge in the purchaser needed to see through the claims made by the seller – are rightly a matter for the courts. We have regulation of insurance products, backed by legislation – rightly.

Forensic statistics

The risk of sudden infant death syndrome (SIDS – "cot death") is well known to all parents – and to those who follow the legal fallout from such human tragedies. One Professor Roy Meadows used to say that one infant death in a family was a tragedy, two was suspicious, three was murder. As a result of such evidence, a number of mothers who had lost more than one child were convicted of their murder and imprisoned. It turned out that the "expert testimony" that led to conviction did not take account of dependence between different deaths in the same family. Professor Peter Donnelly famously described Meadows' evidence in court as "Just plain wrong" (which again I hope will find its place in the Oxford Dictionary of Quotations). One doesn't even need specialist knowledge of either medicine or of probability and statistics to see this. A little knowledge of history will do. Queen Anne was the last of the Stuarts – because, although she valiantly bore seventeen children, they all died, for the same reason (genetic incompatibility between her and her husband).