MATHEMATICAL FINANCE AFTER THE 2007/08/.../13/... FINANCIAL CRISIS

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First, it is a great pleasure to be back here. I thank SoMaS, and in particular my old friend and former colleague Dave Applebaum, for their kind invitation.

When asked to talk, I like to give my hosts a choice of at least two (and often three) alternatives. I offered two this time, one straight mathematics, judged fine for a departmental talk but perhaps too specialised for a general mathematical audience, and one I proposed off the cuff – the present title. This has some mathematics in it. But the content is more concerned with the "big picture" – society in general, and how we in mathematics can help (rather than, as is often perceived and sometimes happens, hinder). So I stand here exposed to two easy charges. The first is that the talk is rather light on mathematics. The second is that the talk has more political content than is usual for SoMaS colloquium talks (indeed, this may be the most political SoMaS colloquium talk ever delivered). To both charges I have an easy defence – indeed, two. The first I have mentioned already – I was asked to speak on this. The second is that this stuff is undeniably important, and is all around us. It is better to talk about the elephant in the room, than to make polite conversation ignoring it, as any Yorkshireman will tell you.

A little mathematical background, from my youth. As a research student, I learned about stochastic processes (a posh term for something random unfolding with time: most things in life are random, and life itself unfolds with time, so these things are all around us). Stochastic processes with stationary independent increments (so what happens in a time-period depends on how long it is, not when it starts, and what happens in one time-period doesn't affect what happens in another if the time-periods don't overlap) are called *Lévy processes*, in honour of the great French probabilist Paul Lévy (1886-1971), who worked out much of their theory in the 1930s. Their behaviour is governed by a *Lévy measure*, which may have infinite mass (just as the line has infinite length). In that case, the process has infinitely many jumps in finite time. I remember learning this in 1967 or so, and thinking that that was lovely mathematics, but had nothing to do with the real world.

Back then, such things were taught by people like me to people like me, in small numbers. Now, such things are taught (still by people like me, unavoidably) to large numbers of eager MSc students studying mathematical finance. They are widely used to model the price evolution of highly liquid, heavily traded stock, under normal market conditions. The lots of jumps, called *jitter*, reflects the lots of trades. Any trade, looked at closely enough, shifts prices: it alters the current balance of supply and demand, and price is the level at which markets clear – supply and demand balance. This is stable enough (under normal market conditions) in broad outline, but all over the place in fine detail. Recall Hooke's *Micrographia* of 1665 (Robert Hooke (1635-1703)), which first revealed the exquisite detail of a fly's eye to an admiring world.

This is a Sheffield talk, ladies and gentlemen, and Sheffield was famous when I was a boy for its steel, and in particular for its cutlery. Hence the Blades as the nickname for Sheffield United Football Club, and hence the title *Steel City Scholars* for the centenary history of this University in 2005. I have two reasons for mentioning the 1997 film *The Full Monty*, famously about a group of unemployed Sheffield ex-steel workers, who try to pay off their debts by forming a group of male strip-tease artists. The first is the upbeat piece of cinema newsreel from two or three decades before, proudly extolling the local steel industry, and then saying "But Sheffield is a city that really knows how to enjoy itself", etc. (the second is the striptease aspect, to which I will return later). One of the things that I am proud of in my British heritage is that Britain was prominent in the Scientific Revolution (Newton's *Principia* of 1687 is the most important book in the history of science: it gave us both calculus and Newton's Laws of Motion), and Britain pioneered the Industrial Revolution. I am thoroughly ashamed of the fact that Britain also pioneered the De-Industrialization Revolution, beginning 1979, throwing a lot of good skilled people on the scrap-heap, writing off a vast amount of capital, and changing over to (and this was before the Crisis) an economy based, not on making things, but on services and the twin pillars of the housing market and the financial services industry, both now understood as bubble-like, and both now burst, as is the way of bubbles.

This by way of background on *economics*. Much of economics is concerned with how prices are arrived at (supply and demand, etc.). Finance in the broad sense is concerned with supply of capital for productive investment (if you're lucky, and for a hostile takeover of a large company, or a football club, if one isn't). Finance was an art rather than a science until the work of

Harry Markowitz (1927-). In his thesis of 1952, he did something that I confess to envy (and I am not given to envy): he wrote a thesis so good that its thrust became a permanent part of the subject (by contrast, even I haven't consulted my thesis of 1969 for at least thirty years). Markowitzian insight 1: *think of risk and return together, not separately.* Markowitzian insight 2: *diversify*, and hold a *balanced* portfolio (lots of negative correlation – so that when things change, and they will, one's losses on the swings will be offset by one's gains on the roundabouts). This led to the *capital asset pricing model* (CAPM – "cap-emm") of the 1960s, already quite mathematical/statistical.

Someone with money to invest has two broad choices: to invest money risklessly, in the bank or building society, say, or by buying Government bonds (let us tactfully pass over bank failures – even the young here remember the Northern Rock panic of 2008, and governments do default – Russia and Mexico did in fairly recent memory, etc.), or to invest in (say) the stock market: invest in risky assets (the company may go bankrupt, when one may lose one's investment) – clearly only sensible if one expects a higher rate of return than on money invested risklessly, and if one has made a sensible comparison between the excess return and the risk involved (à la Markowitz). Imagine one's granny gives one a thousand pounds, and tells one to invest it, for a year. One selects one's candidate stocks for purchase – and is then smitten with cold feet. What if a stock one has bought goes down, rather than up? (I mean in real terms – discounted to allow for interest; we can all do the mathematics of compound interest, which is just exponential growth, so enough of that.) One will then be sorry one bought it; if it goes up, one will be glad. Wouldn't it be nice if one's Fairy Godmother appeared, and gave us a piece of paper, that said that if we were glad we'd bought it, we did buy it, but if we were sorry, we didn't. This would be nice to have: it couldn't lead to a loss, but might lead to a profit. Now Fairy Godmothers do not exist, but such pieces of paper do: they are called *options*, and are the prime examples of *financial derivatives* – financial products derived from something underlying – here the stock price of the company.

What is such an option worth? For many years it was thought that there could be no answer to this question: it would necessarily depend on the attitude to risk of the economic agent (in technical terms, on his *utility function*). It turns out that, under assumptions (admittedly an over-simplification of reality, but still useful, at least to a first approximation), it doesn't. This is because the option can be *replicated*: it is financially equivalent to a suitable combination of cash and stock. Any fool can price that (count the

cash; count the stock; look up the stock price; do the arithmetic) – and the answer is *the same for everyone*. This insight emerged in 1973, when the *Black-Scholes formula* for options was published (Fischer Black (1938-1995) and Myron Scholes (1941-); also in 1973 Robert Merton (1944-) gave a more direct approach; meanwhile, 1973 was also the year when the first exchange for buying and selling options opened, the Chicago Board Options Exchange (CBOE)).

The Black-Scholes formula changed the world. Because for the first time options could be valued, or priced, they could be traded, like any other commodity. This led to an explosive growth in volume of options traded. Often, before the Crash, the size of the market in derivatives exceeded that of the market in the underlying, by an order of magnitude or more. This led to an increasing divergence between the real world, of the real economy and people who make things (Sheffield, steel or whatever), and the financial world, of people who create and trade in pieces of paper, or nowadays, their electronic equivalent. An artificial world that outgrows its natural size becomes unstable, and it is hardly surprising that all this ended in tears, as we now know it did.

Back to mathematics for a while: the stochastic process relevant to the Black-Scholes formula is *Brownian motion*. This has a fascinating history, but takes its name from Robert Brown's observation of 1928 of pollen particles in perpetual random motion, observed under a microscope in suspension in water. This is Brownian motion in the liquid phase, but it was observed in antiquity in the gaseous phase. Lucretius (Cara, c.99 – c. 55 BC), in *De rerum natura*, observed dust particles dancing in sunbeams. The driving force here – heat – only emerged in the 19th century. Louis Bachelier (1870-1946) first put mathematics to work on finance in his 1900 thesis *Théorie de la spéculation* (another thesis that became a permanent part of its subject), where he used *Brownian motion* as a model for the driving noise in the price of a risky asset; he is accordingly now regarded as the founding father of mathematical finance. This was remarkable, as the relevant mathematics did not exist until Norbert Wiener (1984-1964) put Brownian motion on a sound mathematical footing in 1923 (it is accordingly now known also as the Wiener process). The very concept of a stochastic process only emerged that decade, in the work of the Russian school (A. N. Kolmogorov (1903-1987); A. Ya. Khinchin (1894-1959)). Later, this led to stochastic calculus, or Itô calculus (Kiyosi Itô (1915-2008) in 1944). Stochastic/Itô calculus (nowadays also known as martingale calculus) turns out to be just the mathematics

needed to do mathematical finance. This goes back to Merton in 1973, but was really nailed down in 1981 by J. M. (Michael) Harrison and S. R. (Steve) Pliska. Look no further for why this material is extensively taught, by people like me, to large numbers of MSc students.

One can put much of the material taught in a nutshell. First, discount everything (allow for interest – work with real prices rather than nominal prices). Next, do an extraordinary thing. Take the real-world probability measure, P say, which models the driving uncertainty out there in the real world (insofar as it affects the price movements of our risky assets). Throw it away, and replace it by a mathematical fiction: a new probability measure, P^*, the equivalent martingale measure (EMM), or risk-neutral measure, equivalent to P in the sense of measure theory (same null sets: the same things are possible or impossible as before), under which *discounted asset prices become martingales*. A martingale is a mathematical model of a fair game. The martingales so obtained are the nice martingales (the closed ones, or uniformly integrable ones). For these, there is one random object – the limit, or the closing value, or the terminal value – and the current value of the martingale is the conditional expectation of this, given current information. As time passes, we gain more information; what we see is *progressive revelation*, as in a striptease (recall *The Full Monty*), with the limit or terminal value playing the role of the finale (recall the closing shot of *The Full Monty*). All this translates exactly to the context of option pricing, with the very word value carrying over from its mathematical sense to its financial sense. The 'bottom line' for a stock option is the *payoff*, at the expiry time (recall our Fairy Godmother earlier). The current option price is the conditional expectation of this payoff, given our current information.

A personal digression – why I am not a big-shot. I am a probabilist, specialising in limit theorems. Calculus (including stochastic calculus) is continuous, and the continuous can be obtained from the discrete by taking a suitable limit. Brownian motion as a model of option pricing just reflects that prices can go up or down, in an unpredictable and seemingly haphazard or random way. This is just the continuous limit of the simplest random situation: coin-tossing. The picture for the relevant approximation is the histogram, showing the binomial distribution (discrete) – crying out to have a nice smooth curve fitted through it. This is the relevant normal (or Gaussian) density curve (continuous). (In passing: C.-F. Gauss (1777-1855) was the greatest of all mathematicians. He gave his name to many things, but the Gaussian density is commemorated on the old (pre-euro) 10DM note. I

love this so much I carry one around with me, and will pass it round during the talk.) When I first heard seminars on Black-Scholes via Brownian motion back in the 70s (the first from my old friend and contemporary Professor M. H. A. (Mark) Davis), I should have put two and two together, as follows. This formula is going places. The mathematics of Brownian motion is too hard for a public as broad as this formula is going to get. Just discretize: any fool can understand coin-tossing. This emerged in 1979, as the famous and ubiquitous Cox-Ross-Rubinstein binomial tree model. Thus passed my six-year exposure to an open goal. Of course, I had plenty else to think about (and did so, productively). I did not sniff the wind enough to detect that the golden age of academia (my golden decade, from 1969, when I entered our chosen profession, to 1979, when HMQ changed her Government) would end, and be replaced by a climate of opinion based on money – not just out there, but here in academia – when being in big demand for reasons to do with big money would give one invulnerability to the pressures we all live with nowadays.

I confessed at the beginning that I would have to mention politics, which has already begun to seep in, so here goes. The financial crisis of my title is important – supremely so (I'm all right, Jack – many are not). Anything important enough becomes political (a dictum I owe to Maurice Couve de Murville (1907-99) (Foreign Minister 1958-68; Premier 1968-69). Politics is not an exact science (Bismarck). Mathematics is an exact science. So there are limits to the extent to which mathematics can help here – but anything important and even partly quantitative can and should be mathematicized as far as possible.

Economics is not an exact science either (how can it be: the issues are so important they are inescapably political); yet, economics is a prime user of mathematics, most obviously via statistics, but also via probability (stochastic modelling) and game theory. One knows when one is right and when one is wrong in mathematics. One doesn't in economics – a subject that is prone to factionalism, to a degree reminiscent of (in some cases worse than) philosophy, literary criticism and the like. There is no general consensus on who was the greatest economist of the last century (there is in Physics – Einstein; discussing Mathematics here would eat up my remaining time!). But there is consensus on who were the two greatest: Keynes (John Maynard Keynes, Lord Keynes (1883-1946)) and either Friedrich Hayek (1899-1992, Nobel Prize 1974) or Milton Friedman (1912-2006; Nobel Prize 1976; U. Chicago 1946-77); I will confine myself to Friedman here. Milton Friedman

was a good statistician when young, and influential on a number of statisticians, including L. J. (Jimmie) Savage. But his most obvious impact on the world was as the leading figure of the "Chicago boys", who came to prominence advising the Pinochet regime after the 1973 coup that overthrew Allende. I have often heard scholars I respect bemoaning that the academic economics profession has been captured by the Chicago boys. They provided the intellectual core of the neo-conservative movement in the US, and the UK. The great change in my life as a citizen came with the elections of Thatcher in the UK in 1979 (already mentioned), and Reagan in the US in 1980. This ended the post-war consensus, which I have looked back on ever since with affectionate nostalgia, as a golden age of vanished innocence. People argued, of course (they always do), but what is striking is how little actually changed back then when political power changed. I cannot resist saying, of the neo-con revolution and its destruction of the post-war consensus: those who did not live before the Revolution can never know how sweet life was (Talleyrand, of the French Revolution).

Keynes is credited with much of the intellectual leadership behind recovery from the Depression of the 1930s, and later from the devastation of WWII. In brief: when economic activity slows or freezes, governments should prime the pump by spending money (Keynes famously suggested building pyramids as better than doing nothing). Again in brief: the Chicago boys believe in free markets, and in trusting free markets – to do better left alone than when meddled with by state interference. If anyone had said in, say, 1980, that within three decades the big car manufacturers of Detroit would be on their knees, and steel would no longer be made in Sheffield, they would have been laughed at. The belief grew that it did not matter how one made money, only that one did so. Plenty of people did so: there was a long asset-price bubble in the 1990s, presided over by Mr Alan Greenspan (1926-), Chairman of the Federal Reserve 1987-06 (under Reagan 1987-88, Bush Senior 88-92, Clinton 92-2000, Bush Junior 00-06; meanwhile "the West won the Cold War", 1989-91). In 2007, Mr Greenspan published his memoirs, *The Age of Turbulence*. The give-away is in the title: he was writing in the calm before the storm, and didn't see the storm coming because storms were not supposed to happen. I was given the book as a present by my wife, so I had to read it; I did so in some trepidation. To my surprise, I thoroughly enjoyed it (indeed, I recommend it); it is well written and informative; the author is clearly highly intelligent. The text was punctuated by odd hints that things can always go wrong and one never knows, but by and large the tone was one

of Panglossian optimism. All is for the best in this best of all possible worlds (Voltaire, *Candide*); markets know best, and are self-correcting, etc. When the roof fell in in the US later in 2007, when the sub-prime mortgage market collapsed, the book sold out, and was re-issued with a postscript, in which Mr Greenspan adjusted his position somewhat. That sold out too; there were several re-printings, each with more adjustment as more of the roof fell in. Mr Greenspan was grilled by the House Committe on Government Oversight and Reform for four hours on 23 October 2008. Then and later he conceded that he had made serious mistakes: "The whole intellectual edifice of risk management has collapsed", and, "I just didn't get it". Quite.

A personal reminiscence about mathematical finance and me: in 1995, I went from being a Professor of Mathematics at Royal Holloway College, London to being a Professor of Statistics at Birkbeck College, London (where I first taught mathematical finance). The interview panel was chaired by the then Master of Birkbeck, Baroness Tessa Blackstone. I sat at one end of the long Council table; she sat at the other – a striking-looking woman, wearing a black dress and with black hair. I kept noticing during the interview that she had prominent black bags under her eyes (which suited her, I thought). She looked as if she hadn't slept for a week – and she probably hadn't. Birkbeck banked with Baring's, a very old bank, also bankers to HMQ. Baring's were effectively bust at that time, having been ruined by one rogue trader, a young man named Nick Leeson (they survived – which is why Birkbeck survived; they were bought out by the Dutch bank ING for one pound sterling). Nick Leeson had been trading on price differences between the Singapore and Osaka stock exchanges. He concealed his losses, for long enough to bring the bank down; meanwhile, the Barings board thought of him as the goose that laid the golden eggs, and thought he had found a clever way to exploit such price movements, *either way*. If anyone on the Barings board had had enough knowledge of physics to know about perpetual motion machines (or rather their impossibility), Maxwell's demon, entropy, the second law of thermodynamics, etc., Barings would still be with us. What do they know of banking and finance, who only banking and finance know.

Let me turn briefly to the life sciences. The great geneticist J. B. S. Haldane (1892-1964) wrote an essay in 1928, *On being the right size*. To those of you who don't know it, here is some homework from this talk: download it, and read it. Everything has its natural size. Obvious evidence of impending trouble before the Crisis of 07-08 was grotesque disproportion: markets in derivatives orders of magnitude bigger than markets in the underlying, etc.

Pre-Crash, much of our national leadership thought that our national economy being disproportionately based on the housing market and the financial services industry, rather than manufacturing a generation earlier, was unimportant/progess/how things were, depending on viewpoint. Germany stuck with manufacturing. When the roof fell in, our economy was left stranded by events; Germany (though having many problems, arising from re-unification, the eurozone, etc.) has coped much better. Again: a generation ago, a county council finance officer who suggested investing the county's cash reserves in an Icelandic bank, because it offered a higher return than British banks, would have been either laughed at or politely no-balled, and told that "if it looked too good to be true, it probably was too good to be true". After decades of Whitehall directives urging local government to be more pro-active and business-oriented, such warning bells as may have been rung were ignored – with consequences we are still living with.

There have been famous asset bubbles in history, such as the Dutch tulip mania (1634-37) and the South Sea Bubble (1719-20) in the UK. One of the most important was the Wall Street bubble preceding the Wall Street Crash of 1929 and the Great Depression in the 30s. This led to political changes – Roosevelt and the New Deal; what really cured the Depression was Pearl Harbour and the massive investment in armaments in WWII. But long before that, banking in the US had been split between "Main Street" and "Wall Street" – ordinary retail/commercial banking and investment banking (often rudely called casino banking nowadays). This was done by the Glass-Steagall Act, passed in the US in 1933. This was repealed in 1999 (Gramm-Leach-Bliley Act), when deregulation was fashionable (the Greenspan/Clinton years).

This brings me to the vexed question of regulation. Older members of the audience will recall that the Bank of England and the Treasury between them handled financial regulation, macro-economic policy, monetary policy, etc. One of Gordon Brown's first acts as Chancellor in 1997 was to transfer fixing bank rate from the Treasury to the Bank, in the hope that this would be seen to de-politicize it, leading to greater confidence and lower long-term interest rates. He also set up a third pillar, the Financial Services Authority (FSA), with a brief for 'light-touch' regulation (the present Government intends to phase out the FSA). But everyone now agrees that banking is too important to be left to bankers. This raises a modern form of Juvenal's question: quis custodiet ipsos custodes (*Satires*; Juvenal, 60-c.130). Who guards the guards? Who polices the police? Who insures the insurers? Who

reinsures the reinsurers? Who is the lender of last resort? (The Bank of England, acting for HMG, whose credit rating has just gone down, etc.)

Recall the Hippocratic oath taken by members of the medical profession: first, do no harm. The various questions all this raises are known as *macro-prudential* issues. There have been a number of concerted attempts at inter-governmental level to make the financial world safer, most notably the Basel agreements, Basel I (1988), Basel II (2004) and Basel III (due 2013). The main aspects here are capital adequacy; stress testing; and market liquidity risk. I well recall the paper by some of my colleagues (including Paul Embrechts of ETH-Zürich) to the effect that Basel II might even make things worse (e.g. by making sure that in a crisis everyone runs in the same direction – one wants them all to run in different directions). Alas, there are fears that Basel III too may prove counter-productive, "by increasing the incentives for banks to game the regulatory framework". This is a sorry state of affairs. It calls to mind a comment of Marx that everyone now accepts: that the greatest danger to capitalism is the behaviour of capitalists. The danger of parasitism is killing the host organism – the world economy, in this case.

Financial crises do occur, and will recur. Broadly speaking, there are two regimes: normal markets, and markets in crisis. Then, assets all fall, diversification doesn't work, liquidity dries up, etc. The mathematics needed here differs from that needed under normal market conditions, discussed earlier. Now, the crucial things are dependence between different assets, and so-called tail-behaviour: the probability of very large losses. This has much in common with the mathematics of the actuarial/insurance/reinsurance industry – a fine profession, doing much socially useful work, which I hope some of the young here present will consider working in.

Finance is the "nervous system" of the economy; it is essential – but an economy dominated by finance is unhealthy. What counts is the checks, balances, controls, regulation etc. I have always been struck by the parallel with the following: the police, and the armed forces, are necessary. But a country dominated by its police is a police state, and one dominated by its armed forces is a military dictatorship. They don't work – and who would want to live in one?

The recent and ongoing crisis has taught us all that we ignore macro-prudential issues at our peril, and get them wrong at our peril. I am merely a humble mathematician, and the cobbler should stick to his last; equally, I am a human being and citizen before I am an academic, and I have been invited to speak on such things. I want to close with two specific recommen-

dations, and a comment.
Recommendation 1: a Tobin tax – also known as a financial transactions tax, or Robin Hood tax. To be levelled at some rate so low as to be trivial to those honest folk who use financial markets when they need to for economically productive reasons, but positive, so to introduce enough friction to scupper the parasitic use of markets for large-scale speculation. The rate of 1 in a thousand has been suggested; proceeds to be used for good public purposes (health, education, infrastructure, poverty relief, foreign aid etc.).
Recommendation 2: a restructuring, by a new version of Glass-Steagall and/or other means, of the banks. There is now consensus that the banks have become "too big to fail". Nothing should be allowed to escape the consequences of its own folly by playing this card. Old hands will recall that Greenspan bailed out Long-Term Capital Management in 1998 (wrongly in my view, then and now), despite the moral hazard involved; Lehman Brothers was allowed to go under (15 Sep 2008). Meanwhile in the UK, Northern Rock (once a safe building society) was rescued from bankruptcy by the Bank of England on 14 Sep 2007, and nationalised 22 Feb 2008 (split 1 Jan, 2010; Virgin 1 Jan 2012).
Comment. One major contributory factor to the Crisis was the grotesque imbalances at geo-economic and geo-financial level. Most obviously: China (the People's Republic of) has had for many years a massive balance-of-trade surplus (traditionally: lack of western political freedoms plus nothing to spend money on). By contrast, the US, from the years of "Reaganomics" on, has run a massive trade deficit. This was financed by Chinese money. From the point of view of the Chinese leadership: there is no point in sitting on one's cash mountain; put it to work – and where better than in the home of capitalism, the US. The sub-prime mortgage crisis in the US resulted from the bursting of their housing bubble (bubbles do burst in the end – we all know that). The housing bubble grew as a result of many factors, of which I mention three:
(i) the climate of de-regulation ("markets are self-correcting, so nothing (much) can go wrong");
(ii) political pressure during the Clinton years to spread the benefits of house ownership more widely (viewed by many as socially progressive at the time);
(iii) aggressive development of new markets (sub-prime housing in this case) by US banks, flooded with hot Chinese money and looking for the best return on it.
We all know what happened next.

I close with some brief comments on *globalization*. I am old enough to remember the days when there were strict limits on how much sterling one could take overseas, on holiday for example. I also remember the Big Bang in the City of London (26 Oct 1986). We have since moved to a world dominated by vast quantities of hot money, able to chase round the globe at the touch of a button in pursuit of higher returns. This is profoundly de-stabilizing; the analogy that always occurs to me is that of a large tanker sailing through dangerous waters – it needs internal bulkheads, to localize damage and prevent contagion or a domino effect. Big money is global; big multinational companies are global. Everything rests on regulation, but regulation is national. Not only that, there is a "race to the bottom", whereby governments (and I include all British governments in the last 30 years here) have sought the favour of multinationals by courting them with favourable tax-breaks and the like. No wonder we have the scandals of Starbucks/Amazon/Google paying derisory taxes in the UK compared to their business base and profits here. No wonder also that the most notorious off-shore tax havens (many of whom, I am ashamed to say, are British dependencies) are awash with mountains of cash, hiding from proper tax and proper accountability. One of the few good things to come out of 9/11 was the US government's discovery that tax havens shield terrorists' money as well that of decent, bona fide tax evaders, so Uncle Sam has developed a belated though very welcome interest in these matters. What will save us in the long run is the realization by governments that we can't go on like this: getting the worst of both worlds, with hot money globalised in enormous volumes and governments attempting to regulate at national level. This will need a preliminary shift in culture and opinion (overdue, but starting to happen), and will be political. But, as before, everything important enough becomes political.

A final plea, to all who teach mathematical finance and all who learn it: it's lovely material, very interesting mathematically, and widely used, so in demand out there. But never forget the big picture. To students: your potential employers won't. Interviewers will know you have passed the relevant exams – otherwise they wouldn't have called you for interview. You can expect to be asked for your views on all the things discussed here and lots more – so you'd better have some.

Thank you.

NHB