M4A32: Vortex Dynamics

Course Outline

The following list of topics will serve as a general basis for the course.

The basics

(i) basic concepts and equations;

(ii) definition of vorticity, circulation;

(iii) Kelvin's circulation theorem; irrotational flow;

(iv) Helmholtz laws of vortex motion;

(v) Biot-Savart representation of velocity field in terms of vorticity field.

Point vortices

(a) dynamics of point vortices and equations of motion;

(b) steady point vortex configurations (rotating/translating vortex pairs; Thomson vortex polygons; von Karman vortex street);

(c) Hamilton's equations and integrability of 3-vortex motion;

(d) stability of equilibria and dynamics;

(e) point vortex dynamics on the sphere.

Vortex patches

(a) definition and equations of motion;

(b) Rankine vortex;

(c) Kirchhoff vortex; Moore/Saffman & Kida generalizations;

(d) limiting patch shapes;

(d) contour dynamics; filamentation;

(e) regularization of point vortex models e.g. arrays of N rotating patches; vortex patch interactions; vortex merger.

Other solutions in two dimensions

(i) Lamb dipole; Chaplygin solutions;

(ii) combined point vortex/vortex patch solutions;

(iii) interaction with solid boundaries; Kirchhoff-Routh path function; conformal mapping;

(iv) more general vortex dynamics on the sphere.

Vortex sheets

(i) Birkhoff-Rott equation;

(ii) Kelvin-Helmholtz instability;

(iii) Moore curvature singularity;

(iv) Kaden spiral and other similarity solutions;

Axisymmetric vortices

(i) Line vortex rings;

(ii) Hill's spherical vortex;

(iii) Vortex rings; speed of travel; leapfrogging;

Other topics:

Vortex filaments; local induction approximation; nonlinear Schrodinger equation; role of viscosity; viscous decay of a line vortex; Burgers' vortex; vortex system of aircraft wing;.....