
M1M1: Problem Sheet 4

Curve sketching

1. Sketch the graph of the function

$$y = \frac{x + 3}{2x + 1},$$

carefully indicating any special features on your graph.

2. Sketch graphs of the functions:

$$(a) \ x^2 - 6x + 8 = 0; \quad (b) \ x \exp(-x^2); \quad (c) \ \frac{\cos x + \sin x}{\sqrt{2}}.$$

3. If

$$y(x) = \frac{x^2 + 2x + 5}{x + 1},$$

show that

$$y(x) = x + 1 + \frac{4}{x + 1}.$$

Hence sketch the graph of the function $y = y(x)$, carefully identifying any asymptotes, stationary points or points of inflexion.

4. Sketch the graph of the function

$$y = \frac{x(x - 2)}{x - 3},$$

carefully indicating any special features on your graph.

5. Sketch a graph of the function

$$y^2 = x(x^2 - 1),$$

carefully indicating any special features on your graph.

6. A curve is given, in polar coordinates, by the formula

$$r = \frac{2}{3 + \cos \theta}.$$

Sketch this curve.

7. A curve is given in parametric form as

$$x(t) = a \cos^3 t; \quad y(t) = b \sin^3 t,$$

where a and b are positive constants. Sketch the curve.

8. Consider the function

$$f(x) = \frac{x^3 - 1}{x^3 + 1}.$$

- (a) Put $f(x)$ in partial fraction form;
- (b) Find and classify all the stationary points of $f(x)$;
- (c) Find all the points of inflexion;
- (d) Sketch the graph of $f(x)$ carefully indicating all the important features of the graph on your sketch (including the stationary points and points of inflexion).