Mathematical Methods **Spring Term 2021**

Problem Sheet 1

1. The function f defined by
$$f(z) = e^z$$
is analytic. Verify that the Cauchy-Riemann equations hold.

2. Determine the real and imaginary parts of $\sinh z$. Use the result to find all complex zeros of $\sinh z$. Similarly, find all zeros of $\cosh z$.

3. Show that the function f defined by $f(z) = z\bar{z}$ is complex differentiable at $z = 0$ but not analytic at $z = 0$.

4. Let $u(x, y) = x^3 + 6x^2y - 3xy^2 - 2y^3$. Using the Cauchy Riemann equations, or otherwise, find an analytic function f with real part $u(x, y)$. Is f unique?

5. Let
$$v(x, y) = \frac{x}{x^2 + y^2}.$$
Use the Cauchy-Riemann equations to find an analytic function f with imaginary part v. Is f an entire function?

6. Prove that an analytic function f is harmonic. That is show that
$$u_{xx} + u_{yy} = 0, \qquad v_{xx} + v_{yy} = 0,$$
where $u(x, y)$ and $v(x, y)$ are the real and imaginary parts of f.