1. (i) Prove that the rooots (real and complex) of the polynomial

$$P(\lambda) = a_0 + a_1\lambda + \ldots + a_{n_1}\lambda^{n-1} + \lambda^n$$

depend continuosly on the coefficients $a_0, \ldots, a_{n-1}$.
(ii) Prove that if λ s a simple root of $P(\lambda)$, then it depends smoothly on $a_0, \ldots, a_{n-1}$.
(iii) Prove that eigenvalues of a matrix depends continuously on the coefficients of the matrix.

2. Consider the system

$$\begin{cases} \dot{x} = -2x + 16y^2, \\ \dot{y} = -z + xy - 4y^2z - 2z^2y, \\ \dot{z} = y. \end{cases}$$

(i) Write down the Taylor expansion of the center manifold up to the terms of order 2 near the equilibrium $(0, 0, 0)$.
(ii) Write down the normal form up to the terms of the third order for the system on the center manifold.
(ii) Is the equilibrium stable or unstable? How many stable periodic orbits can be born at the bifurcations of this equilibrium?

3. Consider the map

$$\bar{x} = a - bx - x^3.$$

(i) Find equations of the bifurcation curves for the fixed points of this map and draw these curves in the plane of parameters (a, b).
(ii) Find equations of the bifurcation curves for the points of period 2 for this map and draw these curves in the plane of parameters (a, b).