Projects for the course "Dynamics of Games"

Supplementary material is on wwwf.imperial.ac.uk/~dturaev

Project 1: Permanent replicator dynamics

Consider the replicator dynamics, $\dot{x}_i = x_i((Ax)_i - x \cdot Ax)$. We say that system is **permanent**, i.e. there exists $\delta > 0$ so that for each x in the interior Δ there exists $T > 0$ so that for $t \geq T$, we have $x_i(t) \geq \delta$ for all $t \geq T$ and $i = 1, \ldots, n$.

Prove the following theorem, which provides a sufficient condition for the system to permanent.

Theorem. *Assume that P is continuous, $P > 0$ on the interior of Δ and for x in the interior of Δ define*

$$\Psi(x) := \frac{\dot{P}(x)}{P(x)}$$

Assume that Ψ extends continuously to the boundary and that $x \in \partial\Delta$,

$$\int_0^T \Psi(x(t))\, dt > 0 \text{ for some } T > 0.$$

Then the corresponding dynamical system is permanent.

Show that this theorem implies that the hypercycle system from section 1.5 in the lecture notes is permanent.

Show that if the replicator system has a unique interior Nash equilibrium $\bar{x}$ then for each x in the interior of Δ we have

$$\frac{1}{T}\int_0^T x(t)\, dt \to \bar{x}.$$

Mastery component: Consider the replicator dynamics of

$$A = \begin{pmatrix} 0 & 0 & 0 & . & . & . & k_1 \\ k_2 & 0 & 0 & . & . & . & 0 \\ 0 & k_3 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & k_n & 0 \end{pmatrix}$$

with $k_i = 1$. Show that this system is permanent.

Project 2: Reciprocity and the success of TFT strategies

Over the last 30 years every year tournaments were held, in which an interative version of the prisoner dilemma (or donation game) (IPR) was played. These tournaments are often called after its founder, Axelrod. In this project you are asked to explore the success of the TFT strategies in these tournaments.

In a few paragraphs, explain the rules of the Axelrod tournements, the various strategies that were used, and the outcomes in the tournement.

Discuss what arguments are given in Sigmund's book, based on evolutionary game theory (and replicator dynamics), of the success of TFT games and explain why in these arguments the success of TFT depends on the environment in which it is used (i.e. on which other strategies are used).

Explain the results from the paper by Press and Dyson.

Why the paper of Press and Dyson has created so much excitement, and discuss to what extend this excitement was warranted?

Mastery component: Include proofs of the results from the paper by Press and Dyson.

Project 3: Learning.

Show that the Blackwell approachability theorem implies Theorem of the 2000 paper by Hart & Mas-Colell.

In the 2003 paper by Hart & Mas-Colell it is shown that there exists no uncoupled dynamics that guarantees converges to Nash equilibria (for general games). Explain and prove this result, in particular explain what the notion of uncoupled dynamics means.

Mastery Component:

In the 2006 paper by Hart & Mas-Colell it is shown that one has uncoupled dynamics which does converge to Nash equilibria. Explain their results in this paper, and why these do not contradict their 2003 paper.

Project 4: Regret learning, a computer science implementation

Implement regret learning to solve Exercise 2.6 in the paper by Neller and Lanctot.

Mastery component: Explain theoretical arguments behind the algorithm.