Mathematical Methods (Spring Term 2021)

Aims
The aim of the module is to study mathematical methods useful in Physics.

Examinable Content
The module is split into four parts:
1. COMPLEX VARIABLES Complex differentiation, analytic functions, Cauchy-Riemann equations, entire functions. Complex integration, Fundamental Theorem of Calculus, Cauchys theorem, Cauchys integral formula and applications (proof of Liouvilles theorem and proof that analytic functions are infinitely differentiable), isolated singularities and Laurent series, poles and essential singularities, meromorphic functions. Residue theorem and application to computing real integrals.

2. FOURIER TRANSFORMS Review of Fourier transforms and Fourier integrals. Properties of the Fourier transform (including Parseval formula and Convolution Theorem). Computation of Fourier transforms using contour integration. Heaviside and sign function, delta function, Fourier transform (and Fourier integral representation) of the delta function. Application of Fourier transforms to solving linear ODEs and PDEs.

3. CALCULUS OF VARIATIONS AND LAGRANGIAN MECHANICS Shortest curve joining two points, Euler-Lagrange equation as a stationarity condition, Lagrangian formulation of mechanics, L = T - V for conservative forces, generalised coordinates and momenta, symmetries and integrals, energy conservation, Hamiltonian formulation of mechanics. Holonomic constraints, Lagrange multipliers.

4. TENSORS Cartesian tensors, tensor algebra, contraction of tensor indices. Vectors and pseudovectors (or polar and axial vectors), Levi-Civita symbol, cross product.

Mathematical Proofs
The examination will predominantly be a test of technical skill rather than the ability to reproduce proofs. However, students are expected to be able to give short proofs (of results given in the lectures and problem sheets or simple results not seen previously). Students will not be asked for any long proofs.