

5. (i) In a binary symmetric channel, where X denotes the digit transmitted and Y denotes the digit received, the following transmissions probabilities hold, with all transmissions independent.

$$\begin{array}{ll} P(Y = 1 \mid X = 1) = 0.9 & P(Y = 0 \mid X = 0) = 0.9 \\ P(Y = 1 \mid X = 0) = 0.1 & P(Y = 0 \mid X = 1) = 0.1 \end{array}$$

The probability of a 1 being transmitted is 0.7.

- (a) Find the probability that a 0 is received.
 - (b) If a 0 is received, find the probability that a 0 was transmitted.
 - (c) If a 5 bit string of all zeros is transmitted, what is the probability that the received string will contain at most one error?
- (ii) In a study to design an email SPAM filter, the following events are defined

- S : email is SPAM
- A_1 : email contains the string “cheapest”
- A_2 : email contains the string “meds”
- A_3 : email contains the string “credit”

It is found that,

$$\begin{array}{lll} P(A_1 \mid S) = 0.2 & P(A_2 \mid S) = 0.4 & P(A_3 \mid S) = 0.2 \\ P(A_1 \mid \bar{S}) = 0.05 & P(A_2 \mid \bar{S}) = 0.1 & P(A_3 \mid \bar{S}) = 0.01 \end{array}$$

Conditional on S assume that A_1 , A_2 and A_3 are independent.

Given that $P(S) = 0.2$, find the probability that the email is SPAM if

- (a) A_1 occurs ($= p_1$, say).
- (b) both A_1 and A_2 occur ($= p_2$, say).
- (c) A_1 , A_2 and A_3 occur ($= p_3$, say).
- (d) Explain why $p_3 > p_2 > p_1$.

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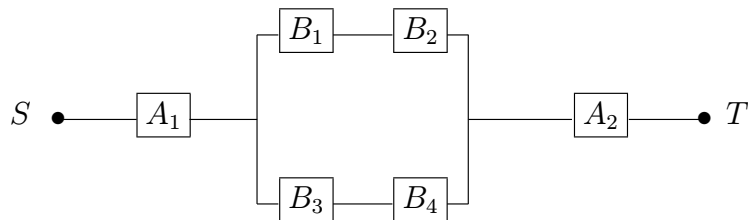
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6. (i) The lifetime, T , of a particular component is normally distributed with mean 6 years and variance 0.25 years^2 .
What is the reliability of the component at 7 years?
- (ii) The lifetimes, T_A and T_B of components of type A and B in hours, have probability density function

$$f(t) = \lambda e^{-\lambda t} \quad t > 0,$$

with $\lambda = 0.1$ and $\lambda = 0.5$ for components A and B respectively.

- (a) Show that $f(t)$ is a valid probability density function.
- (b) Determine the reliability functions and hazard rates associated with T_A and T_B .
- (c) Determine the reliability of each type of component at 90 minutes.
- (d) A system is made up of six components, A_1 and A_2 , of type A and B_1 , B_2 , B_3 and B_4 , of type B . All components operate independently and each have lifetimes as described above. The system functions as long as there a path of functioning components between S and T .



Determine the reliability of the system at 90 minutes.

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