5. (i) (a)

$$\begin{aligned} T_1 &\sim N(2, 0.3^2) \;\Rightarrow\; Z = \frac{T_1 - 2}{0.3} \sim N(0,1) \\ \mathrm{P}(T_1 > 2.6) &= \mathrm{P}\left(Z > \frac{2.6-2}{0.3}\right) \\ &= \mathrm{P}(Z > 2) = 1 - \Phi(2) = 1 - 0.977 = 0.023. \end{aligned}$$

$\boxed{2}$

(b)

$$\begin{aligned} T_2 &\sim N(3, 0.4^2) \;\Rightarrow\; Z = \frac{T_2 - 3}{0.4} \sim N(0,1) \\ \mathrm{P}(T_2 > 4) &= \mathrm{P}\left(Z > \frac{4-3}{0.4}\right) \\ &= \mathrm{P}(Z > 2.5) = 1 - \Phi(2.5) = 1 - 0.994 = 0.006. \end{aligned}$$

$\boxed{2}$

(ii) (a) Let $\overline{t_1}$ be the sample mean for file 1. The 95% confidence interval for μ_1 is

$$\begin{aligned} \left(\overline{t_1} - 1.96\frac{\sigma_1}{\sqrt{n}}, \overline{t_1} + 1.96\frac{\sigma_1}{\sqrt{n}}\right) &= \left(2.01 \pm 1.96\frac{0.3}{\sqrt{50}}\right) \\ &= (2.01 - 0.083, 2.01 - 0.083) \\ &= (1.927, 2.093) \end{aligned}$$

$\boxed{3}$

Let $\overline{t_2}$ be the sample mean for file 2. The 95% confidence interval for μ_2 is

$$\begin{aligned} \left(\overline{t_2} - 1.96\frac{\sigma_2}{\sqrt{n}}, \overline{t_2} + 1.96\frac{\sigma_2}{\sqrt{n}}\right) &= \left(3.97 \pm 1.96\frac{0.4}{\sqrt{50}}\right) \\ &= (3.97 - 0.111, 3.97 + 0.111) \\ &= (3.859, 4.081) \end{aligned}$$

$\boxed{3}$

(b) The CI for μ_1 agrees with the assumed value in part (i), while the CI for μ_2 does not contain the value 3 - the data do not support this value. $\boxed{1}$

(c)

$$\begin{aligned} T &= T_1 + T_2 \\ \mathrm{E}(T) &= \mu = \mu_1 + \mu_2 \\ \mathrm{var}(T) &= 0.3^2 + 0.4^2 = 0.5^2 \\ \Rightarrow T &\sim N(\mu_1 + \mu_2, 0.5^2) \end{aligned}$$

$\boxed{2}$

(d) The 95% confidence interval for μ is

$$\begin{aligned} &\left(2.01 + 3.97 - 1.96\frac{0.5}{\sqrt{50}}, \qquad 2.01 + 3.97 + 1.96\frac{0.5}{\sqrt{50}}\right) \\ &= (5.98 - 0.1386, 5.98 + 0.1386) \\ &= (5.841, 6.119) \end{aligned}$$

$\boxed{4}$

(e) Under the assumption in part (i)

$$\begin{aligned} \overline{T} &\sim N\left(2+3, \frac{0.5^2}{50}\right) \\ \Rightarrow \mathrm{P}(\overline{T} > 6.119) &= \mathrm{P}\left(Z > \frac{6.119 - 5}{0.5/\sqrt{50}}\right) \\ &= 1 - \Phi(15.8) = 1 \end{aligned}$$

this does not agree with the CI in part (ii)(d), the recorded downloading times cast doubt on the assumed values. $\boxed{3}$

6. (i) (a)

$$
\begin{aligned}
R(t) &= \int_t^\infty \lambda e^{-\lambda s}\,ds = \left[-e^{-\lambda s}\right]_0^\infty = e^{-\lambda t}\\
h(t) &= \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda.
\end{aligned}
$$

$\boxed{4}$

For T_A:

$$R_A(t) = e^{-0.2t}; \quad h_A(t) = 0.2.$$

$\boxed{1}$

For T_B:

$$R_B(t) = e^{-0.5t}; \quad h_B(t) = 0.5.$$

$\boxed{1}$

(b)

$$R_A(2) = e^{-0.4} = 0.670; \quad R_B(2) = e^{-1} = 0.368.$$

$\boxed{2}$

(c)

$$
\begin{aligned}
\mathrm{P}(T>2) &= \mathrm{P}(T>2\,|\,A)\mathrm{P}(A) + \mathrm{P}(T>2\,|\,B)\mathrm{P}(B)\\
&= 0.670 \times 0.9 + 0.368 \times 0.1 = 0.640.
\end{aligned}
$$

$\boxed{2}$

(d)

$$
\begin{aligned}
\mathrm{P}(A\,|\,T>2) &= \frac{\mathrm{P}(T>2\,|\,A)\mathrm{P}(A)}{\mathrm{P}(T>2)}\\
&= \frac{0.670 \times 0.9}{0.640} = 0.942.
\end{aligned}
$$

$\boxed{3}$

(ii) (a) Let T be the lifetime of the system

$$
\begin{aligned}
\mathrm{P}(T>2) &= R_B(2)\left(1-(1-R_B(2))^2(1-R_A(2)^3)\right)\\
&= 0.368(1-(1-0.368)^2(1-0.670^3)) = 0.265.
\end{aligned}
$$

$\boxed{4}$

(b) If B_1 and B_2 were replaced with a single component of type A then

$$
\begin{aligned}
\mathrm{P}(T>2) &= R_B(2)\left(1-(1-R(A))(1-R_A(2)^3)\right)\\
&= 0.368(1-(1-0.670)(1-0.670^3)) = 0.283.
\end{aligned}
$$

$\boxed{3}$

So, B_1 and B_2 can be replaced without decreasing the reliability.