5. (i)

$$P(X < x) = \int_0^x \lambda e^{-\lambda x} = \left[-e^{-\lambda x}\right]_0^x = 1 - e^{-\lambda x} = 1 - e^{-0.1x}.$$ 3

(ii)

$$P(\text{excellent}) = P(X < 10) = 1 - e^{-0.1\times 10} = 0.6321,$$ 1

$$\begin{aligned} P(\text{good}) &= P(10 < X < 30) = P(X < 30) - P(X < 10) \\ &= 1 - e^{-0.1\times 30} - (1 - e^{-0.1\times 10}) = 0.3181, \end{aligned}$$ 2

$$P(\text{weak}) = P(X > 30) = 1 - P(X < 30) = e^{-0.1\times 30} = 0.0498.$$ 1

(iii) Let S = event that the file is successfully downloaded.

$$\begin{aligned} P(S) &= P(S \mid \text{excellent})P(\text{excellent}) + P(S \mid \text{good})P(\text{good}) \\ &\quad + P(S \mid \text{weak})P(\text{weak}) \\ &= 1.0 \times 0.6321 + 0.9 \times 0.3181 + 0.1 \times 0.0498 = 0.9234. \end{aligned}$$ 4

(iv)

$$\begin{aligned} P(\text{excellent} \mid S) &= \frac{P(S \mid \text{excellent})P(\text{excellent})}{P(S)} \\ &= \frac{1.0 \times 0.6321}{0.9234} = 0.6845. \end{aligned}$$ 2

(v) Let Y be the number of successfully downloaded files out of n attempts. Then $Y \sim \text{Bin}(n, 0.9234)$ and $P(Y = n) = (0.9234)^n$.

$$\begin{aligned} P(Y = n) = (0.9234)^n &> 0.5 \Rightarrow n \log(0.9234) < \log(0.5) \\ \Rightarrow n &< \log(0.5)/\log(0.9234) = 8.6977 \end{aligned}$$

The maximum number of files is 8. 4

(vi) Let F = event that the first unsuccessful download occurs at the nth download

$$\begin{aligned} P(F) &= P((n-1)\text{successful}) \times P(n\text{th unsuccessful}) \\ &= (0.9234)^{n-1}(1 - 0.9234) = (0.9234)^{n-1}0.0766. \end{aligned}$$ 3

6. (i) Let μ_A and μ_B be the mean lifetime of components of types A and B respectively.
The 95% CI for μ_A is

$$\left(\bar{x}_A \pm 1.96\sigma_A/\sqrt{n}\right) = (26 \pm 1.96/2) = (25.02, 26.98).$$ 3

The 95% CI for μ_B is

$$\left(\bar{x}_B \pm 1.96\sigma_B/\sqrt{n}\right) = (30 \pm 1.96 \times 3/4) = (28.53, 31.47).$$ 3

(ii) Reliability:

$$R(t) = \mathrm{P}(T > t) = \mathrm{P}\left(\frac{T-\mu}{\sigma} > \frac{t-\mu}{\sigma}\right) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$ 2

Hazard:

$$\begin{aligned} h(t) = \frac{f(t)}{R(t)} &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right\} \Big/ \left(1 - \Phi\left(\frac{t-\mu}{\sigma}\right)\right) \\ &= \frac{\sigma^{-1}\phi((t-\mu)/\sigma)}{1 - \Phi((t-\mu)/\sigma)} \end{aligned}$$ 2

(iii)

$$R_A(24) = 1 - \Phi\left(\frac{24-26}{2}\right) = 1 - \Phi(-1) = \Phi(1) = 0.841.$$ 2

$$R_B(24) = 1 - \Phi\left(\frac{24-30}{3}\right) = 1 - \Phi(-2) = \Phi(2) = 0.977.$$ 2

(iv) Potentially gives negative lifetimes! 1

(v) Let N = event that the network is functioning after 24 hours, A and B_1, B_2, B_3, B_4 be the events that individual components are functioning after 24 hours, and $R_N(24)$ = the reliability of the network at $t = 24$ hours:

$$\begin{aligned} N &= A \cap ((B_1 \cap B_2) \cup (B_3 \cap B_4)) \\ \Rightarrow R_N(24) &= R_A(24)\left(R_B^2(24) + R_B^2(24) - R_B^4(24)\right) \\ &= 0.841\left(2 \times 0.977^2 - 0.977^4\right) \\ &= 0.839. \end{aligned}$$ 5