# M3S4/M4S4: Applied probability: 2007-8
# Problems 1: Introduction

1. If $X$ is a continuous random variable which can take only positive values, prove that
$$\mathrm{E}(X) = \int_0^\infty [1 - F(x)]\ \mathrm{d}x.$$

2. Let $T_i$, $i = 1, \ldots, n$ be independent exponentially distributed random variables with parameters $\lambda_i$. Find the distribution of $T = \min(T_i)$.

3. What is the distribution of $T_k$, the number of trials after the $(k-1)$th success up to and including the $k$th success in a series of Bernoulli trials?

4. In a simple birth process with individual birth rate $\beta$ , which begins with one individual, what is the mean of the distribution of $T_n$, the time between the $(n-1)$th and $n$th birth?

5. Suppose we have a simple birth process, with parameter $\beta$ . Using the deterministic approximation, derive a differential equation for $x(t)$, the size of the population at time $t$, and solve this equation given that $x(0) = 1$.

6. A student is learning to juggle. The probability of dropping a ball in the interval $[t, t + \delta t]$ is
$$\frac{5}{1 + 10t}\delta t + o(\delta t).$$
Use the deterministic approach to estimate the expected number of drops over
   (a) the first hour of practice,
   (b) the 5th hour of practice.

7. In a particular religious group the surname is passed on through the male offspring only. The number of sons a man has is a random variable, taking values $0, 1, 2, \ldots$ and each man reproduces independently of each other. However, at each generation, new male members join the group and, on average, $b$ new members adopt each existing surname.
   (a) For the deterministic model, in which it is assumed that each man produces $s$ sons, write down a difference equation connecting the number possessing a given surname at the $n$th generation with the number possessing that surname at the previous generation.
   (b) Solve this equation if $x_0 = 1$ .