

1. It has been found in the past that 4% of screws produced in a certain factory are defective. A sample of 10 is drawn randomly from each hours production and the number of defectives is noted. In what fraction of these hourly samples would there be at least 2 defectives? What doubts would you have if a particular sample contained 6 defectives?

2. A compacted subgrade is required to have a specified density of 110kgs/m<sup>3</sup>. It will be accepted if at least 4 out of 5 cored samples have the specified density.

Assuming that each sample has a probability of 0.8 of meeting the requirements, what is the probability that the subgrade will be acceptable.

3. In the production of copper wire, imperfections are known to occur on average once every 200m. of wire. Assuming that the number of imperfections in a given length of wire has a Poisson distribution, what is the probability that a randomly selected length (i) 200m. long, and (ii) 400m. long, contains no imperfections?

If 4 lengths, each 400m. long, are selected at random, what is the probability that only one of them is free of imperfections?

4. Suppose that telephone calls have random lengths (in minutes) with exponential density

$$f(x) = \frac{1}{3} \exp\left(-\frac{x}{3}\right) \quad x > 0.$$

Let  $X$  denote the length of a call.

- (a) Find  $P(X < 2)$ .
  - (b) Find the probability that a call lasts between 4 and 7 minutes.
  - (c) Find the probability that a call exceeds 4 minutes.
  - (d) What is the conditional probability that a call lasts less than 7 minutes, given that it has already lasted 4 minutes?
  - (e) What is the mean length of call?
5. In the manufacture of a steel rod to fit into an assembly, we assume that the outer diameter (OD) is approximately normally distributed with the mean equal to 2.30cm. and the standard deviation equal to 0.06cm. The specification limits are  $2.31 \pm 0.10$ cm. A piece with OD below the lower specification is considered scrap, while an OD above the upper specification can be reworked. We want to know
  - (a) What percentage of scrap is being produced.
  - (b) What percentage is reworkable, and
  - (c) The maximum percentage we could produce with as little scrap and rework as possible by moving the mean OD to 2.31cm.

6. At one stage in the manufacture of an article a piston of circular cross-section has to fit into a similarly shaped cylinder. The distribution of diameters of pistons and cylinders are known to be normal with parameter:

Piston diameters: mean 10.42cm. standard deviation 0.03cm.

Cylinder diameters: mean 10.52cm. standard deviation 0.04cm.

If the pistons and cylinders are selected randomly for assembly, what proportion of pistons will not fit into cylinders? What is the chance that in 100 pairs, selected at random,

- (a) all the pistons will fit,
- (b) less than 2 of the pistons will fail to fit?

7. The number  $X$  of granite pebbles in specimens of equal size from a beach is thought to follow a Poisson distribution with some unknown parameter  $\mu$ . Estimate  $\mu$  from the following data on 100 pebble specimens.

Frequency distribution of the number ( $X$ ) of  
granite pebbles in 100 specimens of beach pebbles

No ( $X$ ) of granite pebbles in specimen	Frequency of specimens with $X$ granite pebbles
0	58
1	33
2	7
3	2
> 3	0
Total	100

By comparing the observed relative frequency distribution with the estimated Poisson probabilities, comment on the adequacy of the Poisson distribution to describe the distribution of number of granite pebbles.