1. The actual amounts of cement which a filling machine from manufacturer M_1 puts into nominally "ten kilogram (Kg)" bags are independently normally distributed with standard deviation 0.05 Kg. There is a supplier criterion that the bags contain more than 9 Kg. The manufacturer wishes to set the mean fill of the bags such that 99% of the bags satisfy the criterion.

   (i) Determing the mean fill which meets this criterion?

   A second manufacturer, M_2, produces bags whose fill in "ten Kg" bags are independently normally distributed with mean 9.5 Kg and standard deviation 0.2 Kg.

   (ii) What percentage of bags from M_2 meet the criterion?

   A supplier stocks 80% cement from M_1 (with the mean fill as determined in part (i)), with the remainder from M_2.

   (iii) Determine the probability that a randomly selected bag of cement from the supplier meets the criterion.

   (iv) If the bag fails to meet the criterion, what is the probability it came from M_1?

   A builder purchases 20 bags of cement at random from the suppier. Determine the probability that

   (v) All of the bags meet the criterion.

   (vi) At most one of the bags does not meet the criterion.

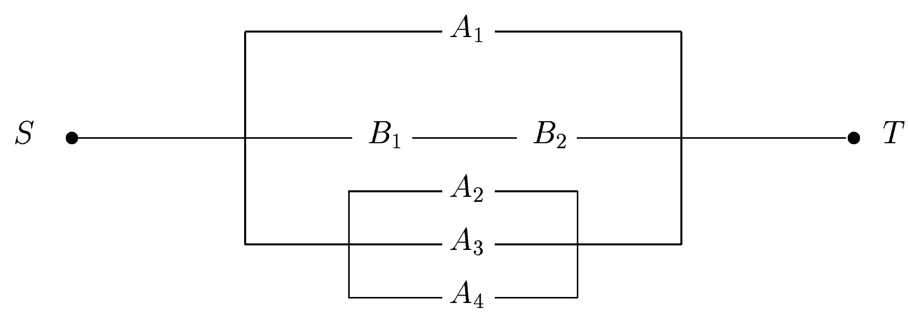
2. (i) The lifetimes, T (in days) of a component of type A and B, follow an exponential distribution,

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

with parameter $\lambda = 0.01$ and 0.02 respectively.

(a) For a component of each type, calculate the probability that it is functioning after 30 days.

A system is made up using four components of type A and two of type B, and functions if there is a path of functioning components between S and T.



(b) Assuming that all components function independently, determine the probability that the system is still functioning after 30 days.

(ii) The lifetimes of a competing system are independently normally distributed. The lifetimes of 20 such systems are measured, giving a sample mean of 34 days and a sample standard deviation of 10 days.

(a) Determine a 95% confidence interval for the mean lifetime of the system.

(b) Without any further calculation, which system (out of the system in part (i) and that in part (ii)) is most likely to be still operating after 30 days?