1. (i) Let X = mean fill, $X \sim N(\mu, 0.05^2)$

$$\begin{aligned}
\mathrm{P}(X > 9|M_1) &= 0.99 \\
\mathrm{P}\left(\frac{X-\mu}{0.05} > \frac{9-\mu}{0.05}\Big| M_1\right) &= 0.99 \\
1 - \mathrm{P}\left(\frac{X-\mu}{0.05} < \frac{9-\mu}{0.05}\Big| M_1\right) &= 0.99 \\
\Phi\left(\frac{9-\mu}{0.05}\right) &= 0.01 \\
\frac{9-\mu}{0.05} &= -2.3263 \\
\mu &= 9 + 2.3263 * 0.05 = 9.1163
\end{aligned}$$

(ii)

$$\begin{aligned}
\mathrm{P}(X > 9|M_2) &= \mathrm{P}\left(\frac{X-9.5}{0.2} > \frac{9-9.5}{0.2}|M_1\right) \\
&= 1 - \Phi(-2.5) = \Phi(2.5) = 0.99379 = 99.379\%
\end{aligned}$$

(iii)

$$\begin{aligned}
\mathrm{P}(X > 9) &= \mathrm{P}(X > 9|M_1)\mathrm{P}(M_1) + \mathrm{P}(X > 9|M_2)\mathrm{P}(M_2) \\
&= 0.99 \times 0.8 + 0.99379 \times 0.2 = 0.9908.
\end{aligned}$$

(iv)

$$\begin{aligned}
\mathrm{P}(M_1|X < 9) &= \frac{\mathrm{P}(X < 9|M_1)\mathrm{P}(M_1)}{\mathrm{P}(X < 9)} \\
&= \frac{0.01 \times 0.8}{1 - 0.9908} = 0.8696.
\end{aligned}$$

Let Y = number of bags that meet the criterion, $Y \sim Bin(20, 0.9908)$

(v)

$$\mathrm{P}(Y = 20) = \binom{20}{20}(0.9908)^20(1-0.9908)^0 = 0.8312.$$

(vi)

$$\begin{aligned} \mathrm{P}(Y \geq 19) &= \mathrm{P}(Y = 19) + \mathrm{P}(Y = 20) \\ &= \binom{20}{19}(0.9908)^19(1 - 0.9908)^1 + 0.8312 \\ &= 0.1544 + 0.8312 = 0.9856. \end{aligned}$$

2. (i)

$$\begin{aligned} \mathrm{P}(T \leq t) &= \int_0^t \lambda e^{-\lambda t_0}\, \mathrm{d}t_0 \\ &= 1 - e^{-\lambda t} \end{aligned}$$

So, $\mathrm{P}(T > t) = e^{-\lambda t}$.

$\mathrm{P}(T > 30|A) = e^{-0.01\times 30} = 0.7408, \qquad \mathrm{P}(T > 30|B) = e^{-0.02\times 30} = 0.5488.$

(ii) Let F = event system functions at 30 days.

$$\begin{aligned} \mathrm{P}(F) &= \mathrm{P}((A_1 \cup A_2 \cup A_3 \cup A_4) \cup (B_1 \cap B_2)) \\ &= 1 - \mathrm{P}(\overline{(A_1 \cup A_2 \cup A_3 \cup A_4) \cup (B_1 \cap B_2)}) \\ &= 1 - \mathrm{P}(\overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cup \overline{A_4} \cap \overline{(B_1 \cap B_2)}) \\ &= 1 - \mathrm{P}(\overline{A_1})\mathrm{P}(\overline{A_2})\mathrm{P}(\overline{A_3})\mathrm{P}(\overline{A_4})\mathrm{P}(\overline{(B_1 \cap B_2)}) \\ &= 1 - (1 - 0.7408)^4(1 - \mathrm{P}(B_1 \cap B_2)) \\ &= 1 - (1 - 0.7408)^4(1 - 0.5488^2) = 0.9968. \end{aligned}$$

(iii) Let T = lifetime, $T \sim N(\mu, 10^2)$. $\bar{x} = 34, s = 10$.
95% CI for μ is

$$\begin{aligned} \mathrm{CI} &= \left(\bar{x} - t^{19}_{2.5\%}\frac{s}{\sqrt{20}}, \bar{x} - t^{19}_{2.5\%}\frac{s}{\sqrt{20}}\right) \\ &= \left(34 - 2.093 \times \frac{10}{\sqrt{20}}, 34 - 2.093 \times \frac{10}{\sqrt{20}}\right) \\ &= (29.3199, 38.6801). \end{aligned}$$

(iv) the system in part (i) is more likely to still be operating after 30 days, as 30 days is included in the 95% CI for the mean in part (ii), implying that at least 2.5% do not last for 30 days, whereas only (100-99.68)% = 0.32% of the systems in part (ii) do not last for 30 days.