OPTIMAL DESIGNS FOR POLYNOMIAL REGRESSION IN ONE VARIABLE

The points of support for the D-optimal design for a dth order polynomial regression model where the design region is [-1, 1] are at $\pm 1$and the roots of the equation

$$P_d^{'}(x) = 0$$

where $P_d(x)$ is the Legendre polynomial of order d. These polynomials are given by

$P_0(x) = 1$
$P_1(x) = x$
$(d+1)P_{d+1}(x) = (2d+1)xP_d(x) - dP_{d-1}(x)$

All points of support have equal weight.

Reference:
Guest, P.G. (1958). The spacing of observations in polynomial regression. Ann. Math. Statist **29**, 294-99.

POINTS OF SUPPORT

d			
2	± 1	0	
3	± 1	$\pm 1/\sqrt{5}$	
4	± 1	0	$\pm\sqrt{3/7}$
5	± 1	$\pm\sqrt{(7 \pm 2\sqrt{7})/21}$	