

# Controlling spatiotemporal chaos in active dissipative-dispersive nonlinear systems

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We present an alternative methodology for the stabilization and control of infinite-dimensional dynamical systems exhibiting low-dimensional spatiotemporal chaos. We show that with an appropriate choice of time-dependent controls we are able to stabilize and/or control all stable or unstable solutions, including steady solutions, traveling waves (single and multipulse ones or bound states), and spatiotemporal chaos. We exemplify our methodology with the generalized Kuramoto-Sivashinsky equation, a paradigmatic model of spatiotemporal chaos, which is known to exhibit a rich spectrum of wave forms and wave transitions and a rich variety of spatiotemporal structures.

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## I. INTRODUCTION

The ability to control a desired particular dynamic state in systems exhibiting chaos, i.e., irregular and unpredictable behavior, is a challenging and fundamental problem in nonlinear science that has attracted considerable attention over the last decades [1]. Chaos and its control are pertinent in a wide variety of natural phenomena and technological applications, from turbulent flows [2], coating processes [3], and reaction-diffusion systems [4] to spatiotemporal instabilities in lasers [5] and cardiac arrhythmias [6], to name but a few. Not surprisingly many different approaches have been proposed to control, up to some extent, different aspects of chaotic dynamics (see, e.g., Ref. [1] for a review on controlling chaos for maps or ordinary differential equations).

Despite the considerable attention that chaos control has received, several important problems have not been resolved. For example, a rigorous and systematic analysis of control of partial differential equations (PDEs) exhibiting low-dimensional spatiotemporal chaos (STC), which is precisely the purpose of our study, is still lacking. Here we consider an important class of PDEs, active dissipative-dispersive nonlinear systems, which are characterized by the presence of coherent structures, the nonlinear interaction of which leads to the emergence of low-dimensional STC. An example of this is the generalized Kuramoto-Sivashinsky (gKS) equation [see Eq. (6) below], which retains the fundamental ingredients of any nonlinear process involving spatiotemporal transitions and pattern formation: nonlinearity, instability or energy production, stability or energy dissipation, and dispersion. Its applications include hydrodynamic thin film instabilities [7], plasma waves with dispersion [8], and step dynamics [9]. Although there have been previous studies on controlling the KS equation (the gKS equation without the dispersion term), e.g., in Ref. [10], they mainly focused on stabilization of the zero solution and for small spatial domains. But it is large domains in spatially extended systems that are typically characterized by the presence of a wide range of characteristic length and time scales which often lead to

complex spatiotemporal behavior. Understanding the precise mechanisms by which low-dimensional STC can be controlled to a desired state, by, e.g., fully controlling the traveling waves of the system, has deserving of attention.

In this study, we present a theoretical framework for stabilizing and/or controlling all stable or unstable solutions, including steady solutions, the position and shape of traveling waves (single and multipulse ones, referred to as bound states), arbitrary periodic functions, and low-dimensional STC. Our framework is based on the application of an appropriate set of time-dependent controls to the system, which are given in terms of the solution we wish to stabilize. We exemplify the methodology with the gKS equation, and we demonstrate both analytically and numerically that its solution can be controlled to any desired (unstable or stable) steady state. This has important consequences for a wide spectrum of applications, e.g., chemically reacting falling films, where controlling the shape of the interface would have profound implications on the associated transport processes [7].

## II. GENERAL METHODOLOGY

Consider infinite dimensional dynamical systems described by PDEs of the form

$$u_t = \mathcal{A}u + \mathcal{D}u + \mathcal{N}(u), \quad (1)$$

where  $\mathcal{A}$  and  $\mathcal{D}$  are a long wave unstable and dispersive linear spatial differential operator with constant coefficients, respectively, and we assume they admit the same set of eigenfunctions.  $\mathcal{N}$  is a nonlinear operator, and the subscript  $t$  denotes time derivative. We consider (1) in a bounded domain with periodic boundary conditions (PBCs) and deterministic initial conditions, i.e.,  $u(x,0) = u_0(x)$ , and for simplicity, assume  $\mathcal{A}$  to be a self-adjoint operator in  $L^2$ , so that its eigenfunctions, denoted as  $\{w_j\}_{j=0}^{\infty}$ , form a basis of  $L^2$ , and we can write  $u(x,t) = \sum_{j=0}^{\infty} u_j(t)w_j(x)$ .

We are interested in controlling general classes of solutions to the gKS equation, including constant solutions, traveling wave solutions or bound states, and nontrivial steady state solutions, denoted as  $\bar{u}$ , which are (linearly) unstable. To stabilize and/or control them, we introduce the following

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controlled equation:

$$u_t = \mathcal{A}u + \mathcal{D}u + \mathcal{N}(u) + \sum_{i=1}^m f_i(t)b_i(x), \quad (2)$$

where  $f_i(t)$  and  $b_i(x)$ , for  $i = 1, \dots, m$ , represent  $m$  controls and  $m$  control actuator functions, respectively. We decompose the solution  $u$  into a stable or unstable system of equations by setting  $u = u_u + u_s$ , where  $u_u = P_N u$  and  $u_s = Q_N u$  are the slow (unstable) and fast (stable) modes, respectively, and  $P_N$  and  $Q_N = I - P_N$  are the corresponding orthogonal projection operators with  $N$  being the number of unstable modes. Equation (2) can then be rewritten as follows:

$$z_t^u = A z^u + D z^u + G + B F, \quad (3)$$

where we have defined  $z^u = [u_u \ u_s]^T$ ,  $F = [f_1(t) \ \dots \ f_m(t)]^T$ , and  $G = [P_N \mathcal{N} \ Q_N \mathcal{N}]^T$ ; and

$$A = \begin{bmatrix} A_u & 0 \\ 0 & A_s \end{bmatrix}, \quad D = \begin{bmatrix} D_u & 0 \\ 0 & D_s \end{bmatrix}, \quad B = \begin{bmatrix} B_u \\ B_s \end{bmatrix},$$

where  $A_u = P_N \mathcal{A}$ ,  $A_s = Q_N \mathcal{A}$ ,  $D_u = P_N \mathcal{D}$  and  $D_s = Q_N \mathcal{D}$ ;  $B_u$  and  $B_s$  are matrices with coefficients  $b_{ji} = \int w_j(x)b_i(x) dx$  for  $j = 0, \dots, N$  and  $j = N+1, \dots, \infty$ , respectively. The key point in our methodology is that whenever there exists a matrix  $K$  such that the eigenvalues of  $A_u + B_u K$  have a negative real part, then a set of controls defined as

$$F = [f_1 \ \dots \ f_m]^T = K(u_u - \bar{u}_u), \quad (4)$$

where  $\bar{u}_u = P_N \bar{u}$ , is able to stabilize the (unstable) solutions  $\bar{u}$  of Eq. (1). The existence of this matrix  $K$  is guaranteed as long as the subsystem described by the matrices  $A_u$  and  $B_u$ , namely,  $u_{u,t} = A_u u_u + B_u F$ , is controllable, which is achievable if the pair  $(A_u, B_u)$  satisfies the Kalman rank condition:

$$\text{rank}[A_u | B_u] = N, \quad (5)$$

where the matrix  $[A_u | B_u]$  is obtained by writing consecutively the columns of the matrices  $A_u^{n-1} B_u$ ,  $n = 1, \dots, N$ . In the following we will rigorously prove this point by using the gKS equation as a model system.

### III. THE GKS EQUATION

Consider

$$u_t + \nu u_{xxxx} + \delta u_{xxx} + u_{xx} + uu_x = 0, \quad (6)$$

normalized to  $2\pi$ -periodic domains  $[x \in (0, 2\pi)]$  using the change of variables  $v = (2\pi/L)^2$ ,  $x = \frac{y}{\sqrt{v}}$ ,  $t = \frac{\tau}{v}$ ,  $\delta = \frac{\delta_L}{\sqrt{v}}$ , and  $u = \frac{u_L}{\sqrt{v}}$ , where  $L$  is the size of the system and  $\delta_L, \tau, y$ , and  $u_L$  are the original parameters, variables, and solution. The parameter  $\delta$  characterizes the relative importance of dispersion so that for  $\delta = 0$  we recover the usual KS equation. It is well known that traveling wave solutions of the KS equation can be unstable, and for sufficiently small values of  $\nu$ , the solutions exhibit chaotic behavior [11–15]. With the addition of dispersion ( $\delta > 0$ ) and for small values of  $\delta$  the dynamics of the gKS equation resembles the KS spatiotemporal chaotic behavior, while sufficiently large values tend to arrest this behavior in favor of spatially periodic traveling waves [16–18]. In a regime of moderate values of  $\delta$ , however, traveling waves

or pulses appear to be randomly interacting with each other giving rise to what is widely known as weak or dissipative turbulence (in the ‘‘Manneville sense’’ [7,19,20]). Our goal is to stabilize and control the traveling wave solutions of (6) in either of these regimes, and hence we have Eq. (2) with  $\mathcal{A}u = -\nu u_{xxxx} - u_{xx}$ ,  $\mathcal{D}u = -\delta u_{xxx}$ , and  $\mathcal{N}(u) = uu_x$ .

Let  $\bar{u}(x, t)$  be a traveling wave of the form  $\bar{u} = U(x - ct)$  where  $c$  denotes the speed of the traveling wave solution, and let  $N = 2l + 1$  be the number of unstable eigenvalues of the linearized KS equation  $u_t = \mathcal{A}u$ . Note that the dispersion term  $\mathcal{D}u$  is antisymmetric in the space  $L^2(0, 2\pi)$ . It has the same eigenfunctions as the KS operator  $\mathcal{A} = -\partial_x^4 - \partial_x^2$ , and, due to antisymmetry, purely imaginary eigenvalues and, in particular, the term  $\langle \mathcal{D}v, v \rangle$  in Eq. (9) below vanishes. Therefore it does not affect the linear stability of the system.

We consider  $u = \bar{u} + v$  to be a solution of Eq. (2), where  $v$  is a perturbation which is described by the following PDE:

$$v_t - \mathcal{A}v - \mathcal{D}v + \nu v_x + (\bar{u}v)_x = \sum_{i=1}^m b_i(x) f_i(t). \quad (7)$$

We wish to prove that  $v$  can be stabilized with an appropriate choice of the controls  $F$ , in particular those defined in Eq. (4). After projecting onto the stable and unstable modes, we obtain that the linearized controlled equation for  $v$  reads

$$\begin{aligned} z_t^v &= \begin{bmatrix} A_u + B_u K & 0 \\ B_s K & A_s \end{bmatrix} z^v + D z^v + E \\ &= C z^v + D z^v + E, \end{aligned} \quad (8)$$

where  $z^v = [v_u \ v_s]$  and  $E = \{P_N[(\bar{u}v)_x] \ Q_N[(\bar{u}v)_x]\}$ .

First we point out that the zero solution to the subsystem  $z_t^v = C z^v$  is exponentially stable if the eigenvalues of  $A_u + B_u K$  have a negative real part (note that by definition  $A_s$  has eigenvalues with a negative real part) [21]. The existence of a matrix  $K$  is guaranteed in our case by construction of the matrix  $B_u$ : it is a square matrix, and since its columns are the discretization of a delta function centered at different points, they are automatically linearly independent. This guarantees that it has full rank, and therefore the Kalman rank condition (5) is satisfied. We can now use a standard Lyapunov argument, as in Refs. [10,22,23], to show that the controls defined in Eq. (4) stabilize the zero solution of the full nonlinear equation (7).

We first make use of the fact that exponential stability of the system  $z_t^v = C z^v$  implies that there exists a positive constant  $a$  such that the operator  $\tilde{\mathcal{A}}v = \mathcal{A}v + \sum_{i=1}^m b_i(x) K_i v_u$ , where  $K_i$  is the  $i$ th row of the matrix  $K$ , satisfies  $\langle \tilde{\mathcal{A}}v, v \rangle \leq -a \|v\|_{L^2}^2$ , and where  $\langle f, g \rangle$  denotes the  $L^2(0, 2\pi)$  inner product. We now define the function  $V(v) = \frac{1}{2} \langle v, v \rangle$  with  $V(0) = 0$  and  $V(v) > 0$  for  $v \neq 0$ . Multiplying Eq. (7) by  $v$  and integrating once we obtain

$$V_t = \langle \tilde{\mathcal{A}}v, v \rangle + \langle \mathcal{D}v, v \rangle - \int_0^{2\pi} v^2 v_x dx - \int_0^{2\pi} (\bar{u}v)_x v dx. \quad (9)$$

Applying PBCs to the above equation we find that the second and third terms of its right-hand side vanish. As for the fourth integral, we have  $-\int_0^{2\pi} (\bar{u}v)_x v dx = -\frac{1}{2} \int_0^{2\pi} \bar{u}_x v^2 dx \leq -\frac{\inf \bar{u}_x}{2} \|v\|_{L^2}^2$  where again we made use of PBCs. Putting things

together we finally obtain

$$\frac{1}{2} \frac{d}{dt} \|v\|_{L^2}^2 \leq - \left( a + \frac{\inf \bar{u}_x}{2} \right) \|v\|_{L^2}^2. \quad (10)$$

We conclude that if the eigenvalues of the matrix  $A_u + B_u K$  are chosen such that  $2a + \inf \bar{u}_x \geq 0$  (note that  $\inf \bar{u}_x < 0$  and hence  $a$  needs to be sufficiently large), we obtain  $V_t \leq 0$ , and so  $V$  is a Lyapunov function for our system, which proves that its zero solution is stable. This choice is possible as long as the pair  $(A_u, B_u)$  satisfies the Kalman rank condition for controllability [21]. Therefore, by using the controls defined in Eq. (4) we can stabilize the traveling wave solution  $\bar{u}$  of the original equation, and the controlled equation is written as

$$u_t - \mathcal{A}u - \mathcal{D}u + uu_x = \sum_{i=1}^m b_i(x) K_i(u_u - \bar{u}_u). \quad (11)$$

It should be noted that with our methodology not only traveling waves but also arbitrary periodic functions, say,  $g(x, t)$ , which are not necessarily solutions of Eq. (6), can be stabilized by adding an extra forcing term as follows:

$$u_t - \mathcal{A}u - \mathcal{D}u + uu_x = \sum_{i=1}^m b_i(x) f_i(t) + \mathcal{L}(g), \quad (12)$$

where  $\mathcal{L}(g) = g_t - \mathcal{A}g - \mathcal{D}g + gg_x$ . The same argument described above for (11) is still valid for (12). For example, we can choose to stabilize the solution to a sinusoidal function  $g(x) = \sin(x)$  for which case we have  $\mathcal{L}(g) = (v - 1) \sin(x) - \delta \cos(x) + \frac{1}{2} \sin(2x)$ .

#### IV. NUMERICAL RESULTS

We look at the numerical solution of the controlled gKS equation (11) for different values of  $\delta$ . In particular, we start by controlling its traveling waves  $\bar{u}(x, t) = U(x - ct) \equiv U(\xi)$  satisfying

$$-cU_\xi + \nu U_{\xi\xi\xi\xi} + \delta U_{\xi\xi\xi} + U_{\xi\xi} + UU_\xi = 0. \quad (13)$$

We solve this equation by making use of a continuation numerical scheme as one of the parameters ( $\delta$  or  $\nu$ ) is varied while the other one is kept fixed. Once we find  $U(\xi)$ , we can construct multipulse initial guesses by taking different solutions centered at different positions, i.e.,  $U_0(\xi) = U(\xi_1) + \dots + U(\xi_n)$ , where  $n$  is the number of pulses we want to control. With this initial guess for Eq. (13) we can find steady bound states (traveling waves) of two or more pulses [18,25].

Time-dependent computations of (11) are performed by making use of a Galerkin truncation up to  $M$  modes for the spatial dependence and a backward differentiation formula of order 2 for time integration [15,26]. Unless specified, the domain size is set to  $L = 20\pi$ , for which there are  $N = 21$  unstable modes, and therefore, we use  $m = 21$  equidistant controls and truncate the system at  $M = 32$  modes. We use point actuator functions, i.e., the functions  $b_i$  are delta functions:  $b_i(x) = \delta(x - x_i)$ . Point actuators are routinely used in engineering applications, for example, in controlling thin-film flows where the gKS equation is applicable. In this case, which is one of the main motivations of our study, point actuators represent liquid that is pumped in or out of the system [10,27,28].

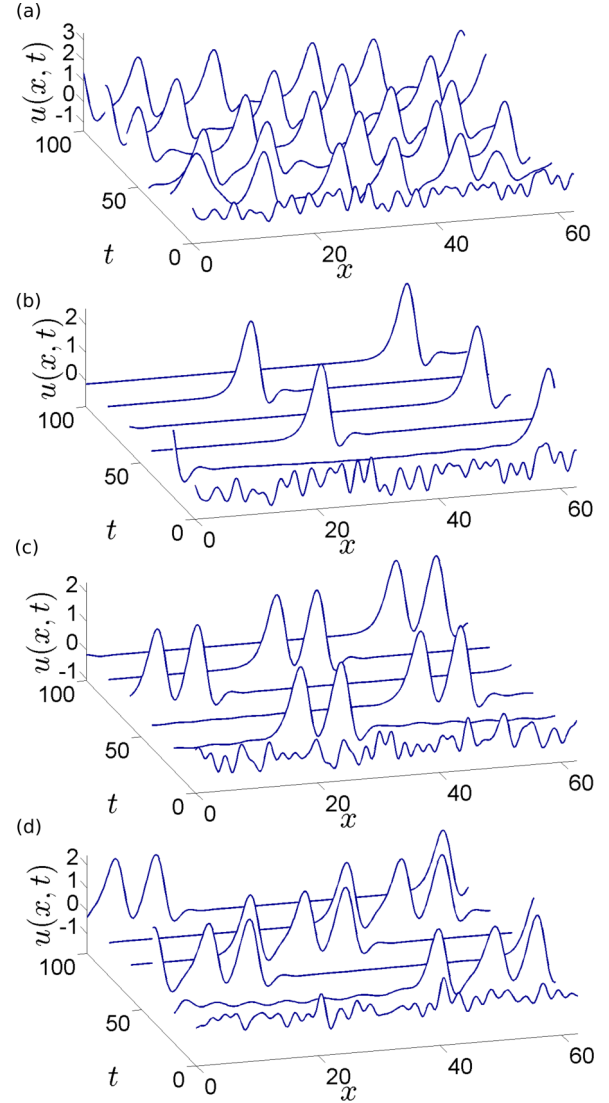


FIG. 1. (Color online) Computations of the gKS equation for  $\delta = 0.5$  (see also “movieDelta05.avi” in Supplemental Material [24]). (a) Uncontrolled spatiotemporal evolution of the gKS solution. Controlled gKS solution to (b) a single pulse, (c) a two-pulse bound state, (d) a three-pulse bound state.

Figure 1 shows the numerical results for  $\delta = 0.5$ , the uncontrolled solution of which is characterized by pulses which are continuously interacting with each other [see Fig. 1(a)], a dynamic state usually referred to as weak or dissipative turbulence [18,19]. With our methodology we can control this chaotic solution to a desired number of pulses traveling as a bound state, as shown in Figs. 1(b)–1(d) where the gKS solution is controlled to a single solitary pulse, a two-pulse bound state, and a three-pulse bound state, respectively. We next look at  $\delta = 0.1$  where the traveling waves are unstable and hence the spatiotemporal dynamics is fully chaotic [see Fig. 2(a)]. Again, we can control the solution to either a single pulse [Fig. 2(b)], a two-pulse bound state [Fig. 2(c)], or a three-pulse bound state [Fig. 2(d)].

A natural and important question is whether the proposed control methodology is robust, in particular with respect to

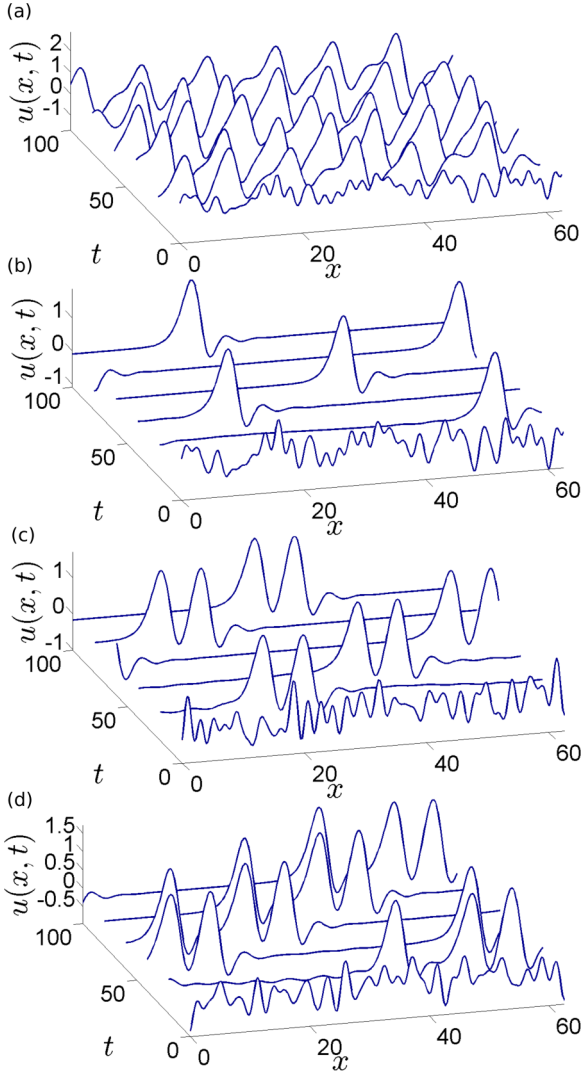


FIG. 2. (Color online) Computations of the gKS equation for  $\delta = 0.1$  (see also “movieDelta01.avi” in Supplemental Material [24]). (a) Uncontrolled spatiotemporal evolution of the gKS solution. Controlled gKS solution to (b) a single pulse, (c) a two-pulse bound state, (d) a three-pulse bound state.

changes or uncertainty in the parameters that appear in the equation, such as  $\nu$  or  $\delta$ . The robustness of our method can be proved rigorously using techniques from control theory, e.g., Ref. [29, Theorem 6]. This analysis will be presented

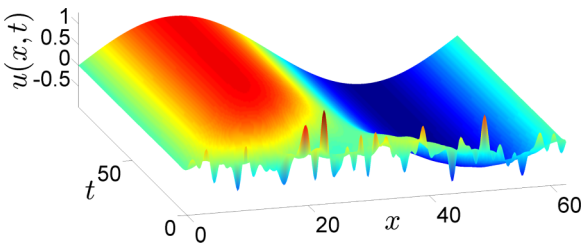


FIG. 3. (Color online) Spatiotemporal evolution of the gKS solution with  $\delta = 0.5$  controlled to the periodic function  $\sin(\frac{2\pi}{L}x)$  (see also “moviePeriodic.avi” in Supplemental Material [24]).

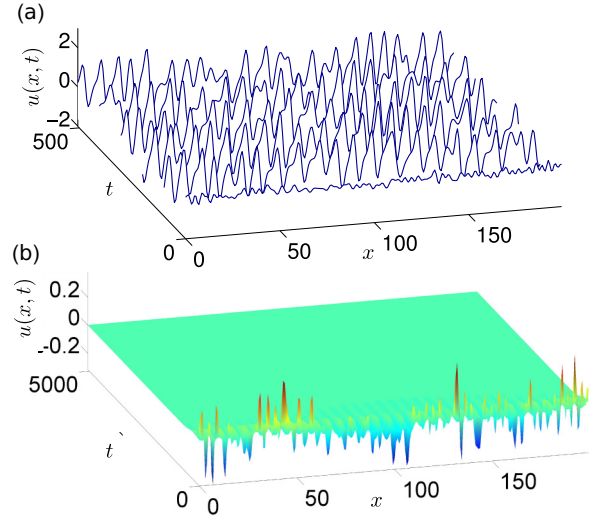


FIG. 4. (Color online) Computations of the KS equation  $\delta = 0$  for the extended domain  $L = 200$  (see also “movieDelta0.avi” in Supplemental Material [24]). (a) Uncontrolled solution. (b) The zero solution is stabilized by using  $m = 63$  controls.

elsewhere. For the purposes of this work, we have performed numerical experiments to test the robustness of the controls, observing that small variations in either  $\delta$  or  $\nu$  do not affect significantly their performance. See “moviePertubDelta.avi” and “moviePertubNu.avi” in the Supplemental Material [24].

As emphasized in the previous section, arbitrary periodic solutions can also be stabilized. Figure 3 shows the gKS solution for  $\delta = 0.5$  forced to evolve as the sinusoidal function,  $\sin(\frac{2\pi}{L}x)$ . We also consider a large domain of  $L = 200$  for  $\delta = 0$ , which supports rather complex chaotic behavior [see Fig. 4(a)], and we control it to the zero solution [see Fig. 4(b)]. Finally, in all computations we measured the energy spent by the controls by using their  $L^2$  norm, which is defined as  $E_1(t) = \sum_{i=1}^m f_i(t)^2$ . The energy of the controls for  $\delta = 0.1$  and  $\delta = 0.5$  is shown in Fig. 5 where it is evident that it rapidly evolves to almost zero; a similar behavior is also observed for  $\delta = 0$  and  $L = 200$  (not shown).

V. CONCLUSION

To conclude, we have presented a generic methodology for controlling (unstable or stable) steady-state solutions and STC in dissipative systems. We have exemplified our methodology with the controlled gKS equation and demonstrated that with

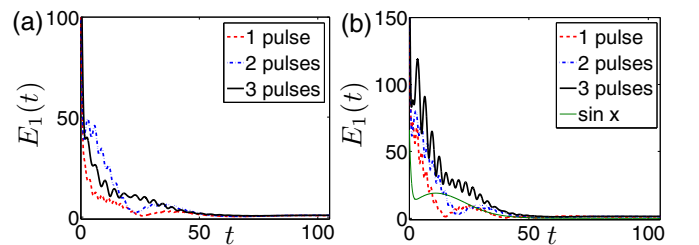


FIG. 5. (Color online) Energy  $E_1(t)$  spent by the controls for  $L = 20\pi$  and for (a)  $\delta = 0.1$  and (b)  $\delta = 0.5$ .

the appropriate choice of controls its solution can be forced to evolve to any desired state, including the unstable zero solution, single traveling waves, bound states of traveling waves for which we can control the number of waves, or arbitrary spatially periodic functions.

We have focused on dissipative systems exhibiting low-dimensional STC; however, the control framework developed here is sufficiently general to allow for its application to a wide spectrum of other nonlinear systems, e.g., reaction-diffusion systems, the control problem of which was studied recently in Ref. [4]. In particular, these authors investigated the control of the position over time of traveling waves of the FitzHugh-Nagumo equation with controls that are proportional to the translational symmetry mode, and, given a prespecified protocol of motion, they obtained an integral equation for the control function. We believe, however, that our framework offers several distinct advantages, since it enables us to control unstable traveling waves and multipulse solutions but also

chaotic behavior in a rigorous and systematic fashion and at a low computational cost. It can also be applied to other types of nonlinear evolution PDEs, such as the Ginzburg-Landau or the KPZ equations. Finally, it should be emphasized that our framework can be readily extended to noisy systems [30], e.g., to controlling the kinetic roughening process of a stochastically growing surface. We believe that our results will motivate further analytical and numerical studies in these directions.

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