Imperial College London

Noncommutative Analysis and Partial Differential Equations

11-15 April 2016

Pure Analysis and PDE Group
http://wwwf.imperial.ac.uk/ ruzh/IC-conference-noncommutative.htm

Organisers:

Chiara Taranto
Durvudkhan Suragan
Julio Delgado
Massimiliano Esposito
Michael Ruzhansky
Nurgissa Yessirkegenov
Rauan Akylzhanov
Thomas Baker
Veronique Fischer

We kindly acknowledge the support of EPSRC, The Taught Course Centre (TCC) and the Department of Mathematics, Imperial College.

1 Abstracts

Group von Neumann algebras and the $L^p - L^q$ boundedness of pseudo-differential operators

Rauan Akylzhanov
Imperial College London, UK
r.akylzhanov14@imperial.ac.uk

(Joint work with Michael Ruzhansky) In this talk, we present a sufficient condition for the $L^p - L^q$ boundedness of pseudo-differential operators on locally compact groups. Our approach is based on the operator algebras techniques. The result depends on a version of the Hausdorff-Young-Paley inequality that we establish for this purpose. In particular, we establish the sufficient conditions for the $L^p - L^q$ boundedness of Fourier multipliers on general locally compact groups. The obtained result also implies the corresponding Hörmander's Fourier multiplier theorem on $\mathbb{R}^n$ and the corresponding known results for Fourier multipliers on compact Lie groups. If time permits, a possible extension to quantum groups will be discussed.

References

[1] L. Hörmander. Estimates for translation invariant operators in L^p spaces. Acta Math., 104:93140, 1960.

[2] R. Akylzhanov, E. Nursultanov, and M. Ruzhansky. Hardy-Littlewood-Paley inequalities and Fourier multipliers on $SU(2)$. arXiv:1403.1731, 2014.

[3] Akylzhanov, R., Nursultanov, E., Ruzhansky, M., 2015. Hardy-Littlewood, Hausdorff-Young-Paley inequalities, and $L^p - L^q$ multipliers on compact homogeneous manifolds, arXiv:1504.07043.

[4] Akylzhanov, R.,Ruzhansky, M., 2015. Fourier multipliers and group von Neumann algebras, preprint.

[5] Akylzhanov, R.,Ruzhansky, M., 2015. Hausdorff-Young-Paley inequalities and $L^p - L^q$ Fourier multipliers on locally compact groups, arXiv:1510.06321.

The Schrödinger equation for the fractional Laplacian on hyperbolic spaces

Jean-Philippe Anker
Université d'Orléans, France
Jean-Philippe.Anker@univ-orleans.fr

(Work in progress with Yannick Sire (Marseille)). We are studying the linear and the nonlinear Schrödinger equation for the fractional Laplacian on real hyperbolic spaces. In this talk we will discuss mainly dispersive estimates and Strichartz inequalities.

Generalised Spherical Functions and the Levy-Khintchine Formula on Groups and Symmetric Spaces

David Applebaum
University of Sheffield, UK
d.applebaum@sheffield.ac.uk

In 1964 Ramesh Gangolli published a Levy-Khintchine type formula which characterised K bi-invariant infinitely divisible probability measures on a symmetric space G/K. His main tool was Harish-Chandra's spherical functions which he used to construct a generalisation of the Fourier transform of a measure. In this talk I will introduce generalised spherical functions (or Eisenstein integrals), and extensions of these which are constructed using representation theory, to obtain such a characterisation for arbitrary infinitely divisible probability measures on a non-compact symmetric space.

Based on joint work with Tony Dooley (Bath, Sydney)

Two classes of Lie groups contractions

Alexis Arnaudon
Imperial College London, UK
alexis.arnaudon@imperial.ac.uk

(Joint work with Rauan Akylzhanov) In this talk we will present a generalization of the Lie group contractions of Inönü and Wigner that contracts a generic Lie group to either a nilpotent Lie group or a semi-direct product of Lie groups. We will then sketch an implementation of these contractions in the unitary dual of a Lie group directly by applying the Borel-Weil theorem to the original group. For this purpose, we assume that it is compact and semi-simple.

References

[1] E. Inönü and E. P. Wigner. On the contraction of groups and their representations. *Proc. Nat. Acad. Sci. U. S. A.*, 39:510–524, 1953.

The Fourier transform on the Heisenberg group: a distribution point of view

Hajer Bahouri
Université Paris-Est Marne-la-Vallée, France
hajer.bahouri@u-pec.fr

In this talk, we want to construct a theory of Fourier transform which can be extended to the tempered distributions on the Heisenberg group. This implies a precise description of the range of the Schwartz space by the Fourier transform.

Banach space representations of nilpotent Lie groups, smooth vectors and pseudo-differential operators

Ingrid Beltiţă
Institute of Mathematics of the Romanian Academy, Bucharest, Romania
ingrid.beltita@gmail.com

We discuss uniformly bounded Banach space representations of nilpotent Lie groups. We then give some results on contragredient representations of Lie groups, with a view toward applications to the abstract characterization of some spaces of pseudo-differential operators. In particular, we give an abstract approach to the description of the norm closure of the pseudo-differential operators of order zero.

Potential Theory results for a class of PDOs admitting a global fundamental solution

Andrea Bonfiglioli
University of Bologna, Italy
andrea.bonfiglioli6@unibo.it

We shall present some potential-theoretic results for a class of hypoelliptic PDOs L admitting a positive and global fundamental solution G(x,y) on N-dimensional space: characterizations of L-harmonic and L-subharmonic functions are available by means of suitable mean-value operators on the level sets of G; by the latter, one can also prove in a simple way the Strong Maximum Principle for L and, in some selected cases, Harnack/Liouville results as well. Due to the relevance of such a global fundamental solution, we present a class of homogeneous (i.e., endowed with dilations) PDOs for which the existence of G(x,y) can be proved via a (global) lifting procedure.

On the algebraic decay of travelling waves

Marco Cappiello
University of Torino, Italy
marco.cappiello@unito.it

Travelling wave-type solutions of nonlinear evolution equations can be often derived as solutions of certain semilinear elliptic equations of the form $p(D)u = F(u)$, where $p(D)$ is a Fourier multiplier and $F(u)$ is a locally bounded nonlinear term. Under suitable assumptions on the symbol $p(\xi)$ of the linear part, it is possible to derive decay estimates for the solutions. In particular, we are interested to the case when the symbol $p(\xi)$ is a sum of homogeneous functions and at least one of them is only finitely smooth at $\xi = 0$. This case occurs in several physical models in fluid dynamics and plasma physics. We derive sharp algebraic decay estimates for the solutions of the equation and we state a precise relation between the smoothness of $p(\xi)$ at $\xi = 0$ and the decay rate at infinity of the solutions. Similar estimates can be proved for the derivatives of the solution when $F(u)$ is smooth. The content of the talk is based on recent results obtained in collaboration with F. Nicola, T. Gramchev and L. Rodino.

References

[1] M. Cappiello, T. Gramchev, L. Rodino, *Decay estimates for solutions of nonlocal semilinear equations*, Nagoya Math. J. **218** (2015), 175-198.

[2] M. Cappiello, F. Nicola, *Sharp decay estimates and smoothness for solutions to nonlocal semilinear equations*, Discr. Cont. Dyn. Systems **36** (2016) n. 4, 1869-1880.

[3] M. Cappiello, F. Nicola, *Pointwise decay estimates and smoothness for semilinear elliptic equations and travelling waves*, Preprint 2016.

Non-Commutative Harmonic Analysis in generalized Clifford Analysis

Paula Cerejeiras
CIDMA, Department of Mathematics, University of Aveiro, Portugal
pceres@ua.pt

The Dirac operator in standard Clifford Analysis describes Fermions as well as in general SU(2)-symmetries. But there is a high interest in physics in the study of SU(n)-

symmetries. This is nowadays mainly modeled by using supersymmetry. But there is another approach to this problem, based on fractional derivatives and higher order decompositions of the Laplacian (see e.g. Herrmann 2005). In this talk we describe fractional Clifford analysis with respect to ternary Clifford algebras. These algebras allow to construct a Dirac operator whose third power is the Laplacian. We begin with the basic tools of a fractional function theory in higher dimensions established by fractional Weyl relations. This enable us to construct a Fischer decomposition and give a full characterization of the spaces of fractional homogeneous monogenic polynomials w.r.t. the fractional Dirac operator. We end this presentation with a description of the group symmetries.

Eigenfunction expansions of ultradifferentiable functions and ultradistributions

Aparajita Dasgupta
École Polytechnique Fédérale de Lausanne, Switzerland
aparajita.dasgupta@epfl.ch

(Joint work with Michael Ruzhansky) In this talk we give a global characterisation of classes of ultradifferentiable functions and corresponding ultradistributions on a compact manifold X. The characterisation is given in terms of the eigenfunction expansion of an elliptic operator on X. This extends the result for analytic functions on compact manifolds by Seeley in 1969, and the characterisation of Gevrey functions and Gevrey ultradistributions on compact Lie groups and homogeneous spaces by the authors (2014).

Schatten-von Neumann properties on compact manifolds

Julio Delgado
Imperial College London, UK
j.delgado@imperial.ac.uk

In this talk we present some recent results on the study of Schatten-von Neumann properties for operators on compact manifolds. A notion of full matrix-symbol on manifolds has been recently introduced in [6] based on a discrete Fourier analysis developed in [3]. We will compare the point of view of full matrix-symbol with the one of kernel. The special case of compact Lie groups is treated separately as an application of the setting introduced in [7]. We will also discuss the case of operators on L^p spaces by using the notion of nuclear operator in the sense of Grothendieck and deduce Grothendieck-Lidskii trace formulas in terms of the matrix-symbol ([1], [2], [4], [5]). (Joint work with Michael Ruzhansky.)

References

[1] J. Delgado *The trace of nuclear operators on $L^p(\mu)$ for $\sigma-$finite Borel measure on*

second countable spaces, Int. Equ. Oper. Theor. 68, p. 61-74, 2010.

[2] J. Delgado *Trace formulas for nuclear operators in spaces of Bochner integrable functions*, Monatshefte für Mathematik. Volume 172, Issue 3, Pages 259–275, 2013

[3] J. Delgado and M. Ruzhansky *Schatten classes on compact manifolds: Kernel conditions*, J. Funct. Anal. 267, no. 3, Pages 772–798, 2014.

[4] J. Delgado and M. Ruzhansky *L^p-Nuclearity, traces, and Grothendieck-Lidskii formula on compact Lie groups*, J. Math. Pures Appl. (9) 102, no. 1, 153–172, 2014.

[5] J. Delgado and M. Ruzhansky *Schatten classes and trace formula on compact groups*, to appear in Math. Res. Lett.

[6] J. Delgado and M. Ruzhansky *Fourier multipliers, Symbols and Nuclearity on compact manifolds*, to appear in J. Anal. Math.

[7] M. Ruzhansky and V. Turunen, Pseudo-Differential Operators and Symmetries: Background Analysis and Advanced Topics, Basel, Birkhäuser, 2010.

Analysis of the minimal representation for pseudo-orthogonal groups

Jacques Faraut
Institut de Mathématiques de Jussieu, Université Pierre et Marie Curie (Paris VI), France
jacques.faraut@imj-prg.fr

The minimal representation of the pseudo-orthogonal group $O(p, q)$ can be realized on a Hilbert space of homogeneous functions on the isotropic cone. This is the Kobayashi-Ørsted model. It can also be realized on a Hilbert space of holomorphic functions on a complex manifold. This is the Brylinski-Kostant model. We will describe a transformation which maps one model to the other. It can be seen as an analogue of the Segal-Bargmann transform.

Wigner measures and effective mass theorems

Clotilde Fermanian-Kammerer
Université Paris Est - Créteil Val de Marne, France
Clotilde.FERMANIAN@cnrs-dir.fr

The dynamics of an electron in a crystal in the presence of impurities is described by a wave function that solves a semi-classical Schrödinger equation where the semi-classical parameter is the ratio between the mean spacing of the lattice and the characteristic length scale of variation of the external potential. Effective Mass Theory consists in showing that, under suitable assumptions on the initial data, the wave function can be approximated in the semi-classical limit thanks to a solution of a simpler Schrödinger equation, the effective

mass equation, which is independent of the semi-classical parameter. It is classical to use in this context Floquet-Bloch decomposition which relies on the spectral theory of periodic Schrödinger operators developed by Bloch in the context of solid state physics. Our goal in this talk is to describe how Wigner measure approach, conjugated with Floquet-Bloch decomposition, can be used to derive effective mass equations. We shall mainly consider two different situations depending on the geometric structure of the set of critical points of Bloch bands: when it consists of isolated points or when it is a submanifold of codimension larger than 1. These results are joint work with Victor Chabu and Fabricio Macia.

Pseudo-differential operators on Lie groups

Veronique Fischer
University of Bath, UK
v.c.m.fischer@bath.ac.uk

In this talk, I will present some recent developments in the theory of pseudo-differential operators on Lie groups.

A survey on weakly hyperbolic equations and systems

Claudia Garetto
University of Loughborough, UK
C.Garetto@lboro.ac.uk

This talk is a survey on some recent work, in collaboration with Michael Ruzhansky (Imperial College London) on weakly hyperbolic equations and systems. The expression weakly refers to the presence of multiple roots/eigenvalues. We will discuss well-posedness for the corresponding Cauchy problem in suitable function spaces and how to deal with low regular coefficients.

Semi-classical analysis of magnetic Schrödinger operators

Bernard Helffer
Université de Nantes, France
Bernard.Helffer@univ-nantes.fr

After Fournais, Helffer, Kordyukov, Morame, Raymond, Sjöstrand, Vu Ngoc ...

Our main object of interest is the Laplacian with magnetic field on a riemannian manifold, but in this talk we will mainly consider, except for specific toy models, a magnetic field

$$\beta = \mathrm{curl}\mathbf{A}$$

on a regular domain $\Omega \subset \mathbb{R}^d$ ($d = 2$ or $d = 3$) associated with a magnetic potential $\mathbf{A}$ (vector field on Ω). Starting from the closed quadratic form Q_h

$$W_0^{1,2}(\Omega) \ni u \mapsto Q_h(u) := \int_\Omega |(-ih\nabla + \mathbf{A})u(x)|^2 \, dx, \tag{1.1}$$

we consider the magnetic Laplacian $\mathcal{H}^D(\mathbf{A}, h, \Omega)$ to be the self-adjoint operator associated to Q_h.

Motivated by various questions in geometry and mathematical physics we consider the following connected problems in the asymptotic $h \to +0$.

Pb 1 Determine the structure of the bottom of the spectrum of the magnetic Laplacian: gaps, typically between the first and second eigenvalue.

Pb 2 Find an effective Hamiltonian which through standard semi-classical analysis can explain the complete spectral picture including tunneling.

Wavelet and Gabor frames on the three-sphere

Uwe Kähler
CIDMA, Department of Mathematics, University of Aveiro, Portugal
kaehler@ua.pt

There is a wide range of applications for function systems on the three sphere, among them X-Ray diffraction tomography. In this application one needs to reconstruct a function on the three-sphere which is well-localized. To approximate such a function we discuss the construction of Wavelet and Gabor frames on the three-sphere. In both cases we use the representation of the corresponding group (Lorentz group in the case of the wavelet transform and Euclidean group in the case of the Gabor transform). While this provides us with the continuous transforms in reality we need a discrete version. To this end we will use coorbit space theory to construct wavelet and Gabor frames. Here we will show the difference in the construction of both cases. To illustrate the applicability of our frames we present an algorithm for the inversion of the spherical X-Ray transform.

Covariant and contravariant calculus of operators on homogeneous spaces

Vladimir V. Kisil
University of Leeds, UK
kisilv@maths.leeds.ac.uk

It is common to study operators by means of certain symbolic calculus, that is, expressing properties of operators in terms of certain functions called their symbols. Symbols may be scalar- or operator-valued functions. The former are usually easier to study, but even the later may provide definite advantages: they resole a single operator into a bundle of simpler ones.

There are various techniques to obtain symbols out of operators and correspondingly decode properties of operators out of their symbols: Simonenko's localisation, Berezin's calculus, Toeplitz operators, PDO calculus, etc. Furthermore, the bilateral connections between functions and operators are usually associated to relations between classical and quantum observables. Thus, there is a physical remake for every mathematical theory in terms of certain (de-)quantisation.

In many (if not all) cases we may find a group, which (at least locally) acts by a non-trivial representation. For example, for operators on a manifold we may bring the action of the Heisenberg group by spatial and phase shifts to every local chart. Thus PDOs, generated according to Roger Howe by the Heisenberg group, become a universal tool for symbolic calculus.

Yet, the universal tool may not be the best adopted to a particular setup. If our operators are related to some other group of symmetries we may prefer to use it instead of the Heisenberg group. Many symbolic methods mentioned above are generated by the affine group, SL(2,R) or certain semi-direct product just in the same way as PDO are generated by the Heisenberg group. We discuss some fundamental results and open problems in this approach.

Onset of instability for a class of non-linear PDE systems

Nicolas Lerner
Institut de Mathématiques de Jussieu, Université Pierre et Marie Curie (Paris VI), France
nicolas.lerner@imj-prg.fr

We consider some quasi-linear systems of PDE and we are looking for ways of detecting some Hadamard instability. The first example that comes up is the elliptic case where the matrix defining the system has a non-real eigenvalue. We explore here the case where the characteristic roots at initial time are real-valued, but leave the real axis for positive time. We prove several instability results and provide examples such as Burgers systems, Van der Waals gas dynamics, and Klein-Gordon-Zakharov systems.

A global quantization for type I locally compact groups.

Marius Mantoiu
Universidad de Chile
mantoiu@uchile.cl

(Joint work with M. Ruzhansky) Let G be a unimodular type I second countable locally compact group and G' its unitary dual. We introduce and study a global pseudo-differential calculus for operator-valued symbols defined on $G \times G'$, and its relations to suitably defined Wigner transforms and Weyl systems. We also unveil its connections with crossed products C^*-algebras associated to certain C-dynamical systems, and apply it to the spectral analysis of covariant families of operators. Applications are given to nilpotent Lie groups, in which case we relate quantizations with operator-valued and scalar-valued symbols.

L^p functional calculus for the Kohn Laplacian on complex spheres

Alessio Martini
University of Birmingham, UK
a.martini@bham.ac.uk

The Kohn Laplacian $\Box_b$ associated to the tangential Cauchy–Riemann complex on a strictly pseudoconvex CR manifold M is a classic example of "non-elliptic Laplacian". In the case M is the unit sphere in $\mathbb{C}^n$, a great amount of information on the spectral theory of $\Box_b$ can be obtained via representation theory. Following this approach we prove a sharp L^p multiplier theorem of Mihlin–Hörmander type for $\Box_b$. In particular, we require on the multiplier a smoothness condition of order $s > (2n-1)/2$, i.e., half the topological dimension of M. It is still an open question whether the same result holds for an arbitrary compact strictly pseudoconvex CR manifold M.

This is partly joint work with V. Casarino (Padova), M. Cowling (Sydney) and A. Sikora (Sydney).

Necessary and sufficient conditions for L^p spectral multipliers on 2-step groups

Detlef Müller
Christian-Albrechts-Universitaet Kiel, Germany
mueller@math.uni-kiel.de

Let G be a 2-step stratified group of topological dimension d and homogeneous dimension Q. Let L be a homogeneous sub-Laplacian on G. By a theorem due to Christ and to Mauceri and Meda, an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on $L^p(G)$ for all $p \in (1,\infty)$ whenever the multiplier F satisfies a scale-invariant smoothness condition of order $s > Q/2$. It is known that, for several 2-step groups and sublaplacians, the threshold $Q/2$ in the smoothness condition is not sharp and in many cases it is possible to push it down to $d/2$. Here we show that, for all 2-step groups and sublaplacians, the sharp threshold is strictly less than $Q/2$, but not less than $d/2$.

This is joint work with Alessio Martini.

Pseudo-differential calculus on Motion Groups

Binh-Khoi Nguyen
Imperial College London, UK
b.nguyen12@imperial.ac.uk

We describe a pseudo-differential calculus on the semi-direct product between V and K, where V is a finite-dimensional vector space and K is any compact connected Lie group acting linearly on V.

On certain a priori estimates for systems

Alberto Parmeggiani
University of Bologna, Italy
alberto.parmeggiani@unibo.it

In this talk I will be concerned in the main (known) positivity estimates for systems: the Sharp-Garding inequality and the Fefferman-Phong inequality. After recalling them, I will discuss their validity in the case of systems and speculate about their nature and generalizations. I will finally discuss the existence of the positive and negative part of certain symmetric 2x2 systems.

Parseval localized frames for subelliptic function spaces on compact manifolds

Isaac Pesenson
Temple University, USA
pesenson@temple.edu

We consider a compact homogeneous manifold $M = G/H$, where G is a compact Lie group and H is a closed subgroup. Let $Y_1, ..., Y_m$, $m \leq n = \dim\ M$, be elements of the Lie algebra $\mathbf{g}$ of G that algebraically generate entire $\mathbf{g}$. Let $X_1, ..., X_m$ be the natural images of the vector fields $Y_1, ..., Y_m$ on the manifold M. We consider the so-called sub-elliptic Laplace operator $\mathcal{L} = X_1^2 + ... + X_m^2$ in the natural space $L_2(M)$ and introduce sub-elliptic Sobolev and Besov spaces associated with $\mathcal{L}$.

The main objective is to construct a frame $\{\phi_\nu\}_{\nu \in J}$ in $L_2(M)$ which has the following properties

1. it is Parseval, i.e.
$$\|f\|^2 = \sum_\nu |\langle f, \phi_\nu \rangle|^2, \ \ f \in L_2(M);$$
2. each ϕ_ν has strong localization on M;
3. each ϕ_ν is bandlimited in the sense of the spectral resolution of $\mathcal{L}$.

Such frame is used to describe relevant Sobolev and Besov sub-elliptic spaces in terms of coefficients $\{\langle f, \phi_\nu \rangle\}_{\nu \in J}$.

Uncertainty inequalities on Lie groups

Fulvio Ricci
Scuola Normale Superiore, Italy
fulvio.ricci@sns.it

In this talk I present several recent developments concerning the Heisenberg-Pauli-Weyl inequality and related inequalities involving sublaplacians.

Decay and regularity of global solutions of elliptic partial differential equations

Luigi Rodino
University of Torino, Italy
luigi.rodino@unito.it

We first survey some results obtained by Gramchev with Cappiello, Rodino, Nicola, Pilipovic and others, concerning decay and regularity of omoclinic solutions of partial differential equations, i.e. global solutions in the Euclidean spaces, tending to zero at infinity. Basic examples are given by the harmonic oscillator of Quantum Mechanics and, in the non-linear setting, by the profile equation for the KdV waves. We then fix attention on the explicit expression of the Weyl symbol of the inverse of the harmonic oscillator (results of Cappiello, Rodino, Toft and Melin) and present some open problems concerning the analytic version of the Shubin operators in Euclidean spaces.

Orthonormal Bases for Square-Integrable Representations of Nilpotent Lie Groups

David Rottesteiner
University of Vienna, Austria
david.rottensteiner@univie.ac.at

Let G be a connected, simply connected nilpotent group and π be a square-integrable unitary irreducible representation modulo its center on $L^2(\mathbb{R}^d)$. We prove that under reasonably weak conditions on G and π there exist a uniform subgroup Γ of G and some compact set $F \subseteq \mathbb{R}^d$ such that

$$\{\mu(F)^{-1/2}\,\pi(\gamma)1_F \mid \gamma \in \Gamma/Z(G)\}$$

forms an orthonormal basis of $L^2(\mathbb{R}^d)$. This construction generalizes the well-known example of Gabor orthonormal bases in time-frequency analysis.

Index Theory for Hyperbolic Operators

Alexander Strohmaier
University of Loughborough, UK
A.Strohmaier@lboro.ac.uk

I will discuss the Atiyah-Singer index theorem, give some example and then explain the Atiyah-Patodi-Singer index theorem for Dirac operators on manifolds with boundary. I will then show that choosing appropriate function spaces and boundary conditions hyperbolic operators such as the Lorentzian Dirac operator on a globally hyperbolic space-time may also be Fredholm. In this case an appropriate analog of the APS index formula holds can be shown to hold. (Joint work with C. Baer).

Hardy and Rellich type inequalities, identities, and sharp remainders on homogeneous groups

Durvudkhan Suragan
Imperial College London, UK
d.suragan@imperial.ac.uk

In this talk we give sharp remainder terms of L^p and weighted Hardy and Rellich inequalities on one of most general subclasses of nilpotent Lie groups, namely the class of homogeneous groups. As consequences, we obtain analogues of the generalised classical Hardy and Rellich inequalities and the uncertainty principle on homogeneous groups. We also present higher order inequalities of Hardy-Rellich type, all with sharp constants. A number of identities are derived including weighted and higher order types. This talk is based on joint works with Michael Ruzhansky.

Sub-Laplacian Gevrey Spaces on the Heisenberg Group and Applications

Chiara Taranto
Imperial College London, UK
c.taranto13@imperial.ac.uk

In our work in progress, we define Gevrey Spaces with respect to the Sub- Laplacian on manifolds. In this talk, we focus on the case of the Heisenberg Group. Keeping in mind the Euclidian case, we show a global characterisation of these spaces on the Fourier transform side. This turns to be fundamental in applications. Indeed we prove the Gevrey well-posedness of the Cauchy problem for the time-dependent wave equation for the sub-laplacian on the Heisenberg group.

References

[1] D. Dasgupta, M. Ruzhansky, Gevrey functions and ultradistributions on compact Lie groups and homogeneous spaces, Bulletin des Sciences Mathématiques, 138 (2014), 756-782.

[2] C. Garetto, M. Ruzhansky, Wave equation for sum of squares on compact Lie groups, J. Differential Equations, 258 (2015), 4324-4347.

[3] M. Ruzhansky, V. Turunen, Pseudo-Differential Operators and Symmetries: Background Analysis and Advanced Topics, Basel, Birkhäuser, 2010.

Hardy's inequality for fractional powers of the sublaplacian on the Heisenberg group

Sundaram Thangavelu
Indian Institute of Science, Bangalore, India
veluma@math.iisc.ernet.in

In this talk we discuss Hardy inequalities for the conformally invariant fractional powers of the sublaplacian on the Heisenberg group H^n. We have two versions of such inequalities depending on whether the weights involved are non-homogeneous or homogeneous. In the first case, the constant arising in the Hardy inequality turns out to be optimal. In order to get our results, we use ground state representations. The key ingredients to obtain the latter are some explicit integral representations for the fractional powers of the sublaplacian and a generalized result by M. Cowling and U. Haagerup. The approach to prove the integral representations is via the language of semigroups. As a consequence of the Hardy inequalities we also obtain versions of Heisenberg uncertainty inequality for the fractional sublaplacian. The talk is based on a joint work with Luz Roncal.

Schatten-von Neumann properties of Weyl operators of Hörmander type

Joachim Toft
Department of mathematics, Linnæus University, Sweden
joachim.toft@lnu.se

Let $t \in \mathbf{R}$ be fixed and consider the *pseudo-differential operators* $\mathrm{Op}_t(a)$ with *symbol* a which is defined by the formula:

$$\mathrm{Op}_t(a)f(x) \equiv (2\pi)^{-n} \iint_{\mathbf{R}^n\times\mathbf{R}^n} a((1-t)x+ty,\xi)f(y)e^{i\langle x-y,\xi\rangle}\,dyd\xi$$

A fundamental result for such operators reads: Assume that $0 \le \delta < \rho \le 1$ and $r \in \mathbf{R}$. Then each $\mathrm{Op}_t(a)$ with $a \in S^r_{\rho,\delta}(\mathbf{R}^{2n})$ is L^2-continuous, if and only if $S^r_{\rho,\delta} \subseteq L^\infty$ (i. e. $r \le 0$). Here recall that $S^r_{\rho,\delta}(\mathbf{R}^{2n})$ consists of all $a \in C^\infty(\mathbf{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{r-\rho|\beta|+\delta|\alpha|}.$$

A somewhat weak property here is that no conclusion concerning L^2-continuity can be done for a *particular* operator $\mathrm{Op}_t(a)$, when $a \in S^r_{\rho,\delta}$ and $r > 0$.

In [1] the theory at this point was performed more complete. For example, if $a \in S^r_{\rho,\delta}$, then it follows from [1] that $\mathrm{Op}_t(a)$ is L^2-continuous, if and only if $a \in L^\infty$.

The theory, which contains the latter result as a special case, is formulated by means of Hörmander-Weyl calculus, where the symbol classes $S(m,g)$ are parameterized with appropriate weight functions m and Riemannian metrics g. The continuity investigations are also performed in a broader context, which involve Schatten-von Neumann properties for such operators. Then it is proved in [1] that the following general result holds true: Assume that $p \in [1,\infty]$, and that the g-Planck's constant h_g satisfies $h_g^N m \in L^p$, for some $N \ge 0$. Then $\mathrm{Op}_t(a)$ is a Schatten-von Neumann operator of order p, if and only if $a \in L^p$.

The previous result was partially recently extended in such way that the Schatten parameter p was allowed to be smaller than 1. More precisely, in [2] it is proved that if $p \in (0,\infty]$, $h_g^N m \in L^p$, for some $N \ge 0$ and $a \in L^p$, then $\mathrm{Op}_t(a)$ is a Schatten-von Neumann operator of order p.

An important example concerns globally defined pseudo-differential operators with symbols in the SG class $\mathrm{SG}^{(\omega)}_{\rho,\delta}(\mathbf{R}^{2n})$, which consists of all $a \in C^\infty(\mathbf{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \le C_{\alpha,\beta}\omega(x,\xi)(1+|x|)^{-\delta|\alpha|}(1+|\xi|)^{-\rho|\beta|},$$

where ω is bounded by a polynomial and $\rho,\delta > 0$. In this case we have that $\mathrm{Op}_t(a)$ is Schatten-p operator, if and only if $a \in L^p$, when $p \ge 1$.

In the talk we explain these results and present some ideas of their proofs.

References

[1] E. Buzano, J. Toft *Schatten-von Neumann properties in the Weyl calculus*, J. Funct. Anal. **259** (2010), 3080–3114.

[2] J. Toft *Continuity and compactness for pseudo-differential operators with symbols in quasi-Banach spaces or Hörmander classes*, arXiv:1406.3820 (2014).

Pseudo-differential operators generated by boundary value problems

Niyaz Tokmagambetov
Al–Farabi Kazakh National University, Almaty, Kazakhstan
niyaz.tokmagambetov@gmail.com

(Joint work with Michael Ruzhansky) We consider the development of pseudo–differential operators generated by boundary value problems. In particular, we derive an explicit formula for the quantization of pseudo–differential operators induced by the derivative operator on a segment. Starts an interesting direction of discrete analysis based on elliptic boundary value problems, continuing, in a sense, the analysis on the torus started by M. Ruzhansky and V. Turunen [1], [2], in which case one may think of a problem having periodic boundary conditions. Some researches on the development of the Fourier analysis based on a non self–adjoint boundary value problem are given in [3]–[5].

References

[1] Ruzhansky M., Turunen V., *Pseudo–Differential Operators and Symmetries*, Birkhauser (2010).

[2] Ruzhansky M., Turunen V., *Quantization of Pseudo–differential Operators on the Torus*, J. Fourier Anal. Appl., **16**, 943-982 (2010).

[3] Ruzhansky M., Tokmagambetov N., *Nonharmonic analysis of boundary value problems*, to appear in Int. Math. Res. Notices; http://arxiv.org/abs/1504.00777

[4] Kanguzhin B., Tokmagambetov N., The Fourier transform and convolutions generated by a differential operator with boundary condition on a segment. In *Fourier Analysis: Trends in Mathematics*, pages 235–251. Birkhäuser Basel AG, Basel, 2014.

[5] Kanguzhin B., Tokmagambetov N., and Tulenov K., Pseudo-differential operators generated by a non-local boundary value problem. *Complex Var. Elliptic Equ.*, 60(1):107–117, 2015.

Theory and applications of time-frequency analysis

Ville Turunen
Department of Mathematics and Systems Analysis, Aalto University
ville.turunen@aalto.fi

When and how often something happens in a signal? By properly quantizing these questions, we obtain the Born–Jordan time-frequency transform, defining a sharp phase-space energy density. We study properties of different time-frequency transforms, and also present computed examples from acoustic signal processing, quantum mechanics and medical sciences.

Operators on compact groups

Jens Wirth
Institute of Analysis, Dynamics and Modelling. University of Stuttgart, Germany
jens.wirth@mathematik.uni-stuttgart.de

In this talk we will present some recent results on the global symbolic calculus of operators on compact Lie groups. Operators on compact Lie groups are described in terms of global symbols, which are matrix-valued functions on the non-commutative phase space $G \times \widehat{G}$, where G denotes the compact Lie group under consideration and $\widehat{G}$ the set of equivalence classes of irreducible unitary representations of the group G. Symbol classes for operators are characterised in terms of differential-difference conditions.

We will focus on the particular example of the 3-sphere $\mathbb{S}^3$, understood as set of unit quaternions, and show how the differential-difference calculus arises in a natural way when considering spaces of homogeneous polynomial as the representation spaces. We will give explicit formulas and recurrence relations and explain central parts of the associated calculus. Applications involve differential operators, the Szegő projector and projections associated to the 2-sphere as homogeneous space of $\mathbb{S}^3$.

2 Posters

On a nonlocal initial boundary value problem for the time-fractional diffusion equation

Gulaiym Oralsyn
Institute of Mathematics and Mathematical Modelling, Almaty, Kazakhstan
g.oralsyn@list.ru

In this work we discuss methods for constructing trace formulae for the heat-volume potentials of the time-fractional diffusion equation to piecewise smooth lateral surfaces of cylindrical domains and use these conditions to construct as well as to study a nonlocal initial boundary value problem for the time-fractional diffusion equations. We also obtain similar results for higher powers of the time-fractional diffusion equations.

3 Partipants

Yermurat Adilbekov (Institute of Mathematics and Math. Modeling, Almaty, Kazakhstan)
Rauan Akylzhanov (Imperial College London, UK)
Jean-Philippe Anker (Université d'Orleans, France)
Dave Applebaum (University of Sheffield, UK)
Alexis Arnaudon (Imperial College London, UK)
Hajer Bahouri (Université Paris-Est-Creteil, France)
Ingrid Beltiţă (Institute of Mathematics of the Romanian Academy, Romania)
Andrea Bonfiglioli (University of Bologna, Italy)
Marco Cappiello (University of Torino, Italy)
Paula Cerejeiras (University of Aveiro, Portugal)
Aparajita Dasgupta (École Polytechnique Fédérale de Lausanne, Switzerland)
Julio Delgado (Imperial College London, UK)
Claudia Garetto (University of Loughborough, UK)
Nicola Garofalo (University of Padova, Italy)
Jacques Faraut (Institut de Mathématiques de Jussieu, Université Paris 6, France)
Clotilde Fermanian-Kammerer (Université Paris Est, France)
Veronique Fischer (University of Bath, UK)
Bernard Helffer (Université de Nantes, France)
Uwe Kähler (University of Aveiro, Portugal)
Aidyn Kassymov (Institute of Mathematics and Math. Modeling, Almaty, Kazakhstan)
Vladimir Kisil (University of Leeds, UK)
Nicolas Lerner (Institut de Mathématiques de Jussieu, Université Paris 6, France)
Remi Lodh (Springer UK)
Marius Mantoiu (University of Chile, Chile)
Alessio Martini (University of Birmingham, UK)
Akhymbek Meiram (Institute of Mathematics and Math. Modeling, Almaty, Kazakhstan)
Detlef Müller (Christian-Albrechts-Universität zu Kiel, Germany)
Gulzat Nalzhuppaeva (Kazakh National University, Almaty)
Binh-Khoi Nguyen (Imperial College London, UK)
Gulaiym Oralsyn (Institute of Mathematics and Math. Modelling, Almaty, Kazakhstan)
Alberto Parmeggiani (University of Bologna, Italy)
Isaac Pesenson (Temple University, USA)
Fulvio Ricci (Scuola Normale Superiore, Pisa, Italy)
Luigi Rodino (University of Torino, Italy)
David Rottensteiner (University of Vienna, Austria)
Alexander Strohmaier (University of Loughborough, UK)
Durvudkhan Suragan (Imperial College London, UK)
Chiara Taranto (Imperial College London, UK)
Joachim Toft (Linnaeus University, Sweden)

Sundaram Thangavelu (Indian Institute of Science, Bangalore, India)
Niyaz Tokmagambetov (Kazakh National University, Kazakhstan)
Ville Turunen (Aalto University, Helsinki, Finland)
Jens Wirth (Stuttgart University, Germany)